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New methodology to obtain free surfaces in unconfined seepage problems. The Gasset Dam (Ciudad Real, Spain).

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ABSTRACT

A finite element couple model to obtain the free surface in unconfined seepage problems is presented. It is a two-dimensional, finite element numerical model formulated in displacements, which allows us to calculate the position of the free surface in steady flow conditions, by changing the impermeability boundary conditions in an iterative manner, keeping constant the domain and the FE mesh. The accuracy in the computation of the free surface position into an earthfill dam, is of paramount importance in its design stage as well as to ensure its safety. Aiming to corroborate the suitability of this new methodology, a study of the Gasset Dam (Ciudad Real, Spain) has been made with success, obtaining the free surface position and the total discharge through the dam.

1. INTRODUCTION

The problem of calculating free surfaces in unconfined seepage media has been dealt with by many geotechnical researchers during the last forty years. It is possible to find in the literature several analytical solutions for particular problems, but they usually involve extremely restrictive assumptions which make these procedures not very useful in real field cases (Harr, 1962). This circumstance makes the numerical procedures almost the only possibility for obtaining feasible solutions when heterogeneous and geometrically complicated porous media are analysed. Hence, finite element based approaches are of general use. The first developed methods of this type consisted of obtaining the free surface in an iterative way, assuming its location, generating a mesh inside the flow domain, obtaining the flow network inside this domain, checking the consistency of the free surface conditions, and thus, locating another boundary. This procedure usually involves a high number of iterations and numerical effort, due to the necessity of remeshing (Taylor and Brown, 1967; Finn, 1967; Neuman and Witherspoon, 1970). Several more advanced methodologies were based on adaptative meshing, in which the free surface is also obtained after an iterative procedure, but moving the nodes located at the free surface, meaning that it is not necessary to create the mesh in each iteration. The main countermeasure in this case is the possibility of

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having highly distorted elements at the end of the computation, producing high numerical errors to appear (Oden and Kikuchi, 1980). More efficient models are those in which the flow domain is constant, but the soil properties (mainly, the permeability) are variable depending on the location of the free surface (Desai, 1976; Bathe and Khoshgoftaar, 1979; Lacy and Prevost, 1987; Borja and Kishnani, 1991). Another possibility consists of keeping constant both flow domain and soil properties, and making the impermeability boundary conditions variable in the requited iterations, to avoid the entrance of fluid to the porous medium through those borders where this circumstance is physically not possible (López-Querol et al, 2011). This methodology has been successfully tested with theoretical examples under steady conditions, but not using real field cases so far, for which data are usually scarce and difficult to find.

The present research shows the application of this new methodology to a real field case: the Gasset dam in Ciudad Real (Spain). This infrastructure is of paramount importance for water supplying the population in a wide zone, as well as for irrigation purposes. This dam was built about a century ago, and it has suffered several renewing works, amongst other reasons, aiming to increase the maximum volume of the reservoir, and to solve several filtrations and leaking problems. Thus, it is a very non homogeneous porous media, with a non easy geometry (Peco and López-Querol, 2012).

The paper begins explaining the mathematical and numerical models, as wells as some details on the code developed ad hoc for analysing this earth dam. After that, the Gasset dam, with all its features, is presented, and the geotechnical parameters as well as the available data are described. After showing the comparison of the numerical model results and field measures under steady flow conditions, a new analysis of the dam before being repaired is also attached, introducing an interpretation of the pathology producing some leaking to appear some decades ago. All these results show the feasibility of this methodology for analysing real field cases.

2. DESCRIPTION OF THE MODEL AND ITERATIVE PROCEDURE

2.1 Mathematical model

The employed mathematical formulation follows the Biot's equations, which govern the transmission of stress waves though saturated porous media (Biot, 1959). The model is formulated according to Zienkiewicz et al. (2000). In a differential, saturated element of soil, consisting of a part or solid phase and water saturating the voids, the equilibrium equation is given by Eq. (1):

$$S^T \cdot D^e \cdot S \cdot \{u\} - S^T \cdot m^T \cdot P_w - \rho \cdot \{\ddot{u}\} - \frac{\rho_w}{n} \cdot \{\ddot{w}\} + \rho \cdot \{b\} = \{0\} \quad (1)$$

The equilibrium of the fluid phase is governed by Eq. (2):

$$-\nabla p_w - K^{-1}\{\dot{w}\} + \rho_w\{b\} - \rho_w\{\ddot{u}\} - \frac{\rho_w}{n}\{\ddot{w}\} = \{0\} \quad (2)$$

Eq. (3) represents the continuity of flow through the saturated porous medium:

$$\dot{p}_w = -Q \cdot (\nabla^T\{\dot{w}\} + m^T\{\dot{\varepsilon}\}) \tag{3}$$

In the above equations, {.} means vectors, u represents the solid phase displacement, w is the liquid phase displacement relative to the solid phase, ρ y ρ_w denote the soil and water densities respectively, b is the vector of external forces (gravity forces), n is the porosity of the solid skeleton, K denotes the matrix of physic permeability whose components are expressed in m3·sec/kg units, p_w is the pore pressure, Q represents the volumetric compressibility of the mixture solid-liquid which is expressed as:

$$Q = \frac{K_w}{n} + \frac{K_S}{1-n} \tag{4}$$

where K_w and K_S are, respectively, compressibilities of the liquid and solid phases.
In a two dimensional, plane strain approach, the matrix operator, S, follows the next expression:

$$S = \begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} \tag{5}$$

and the vector m, is defined as:

$$m = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{6}$$

The constitutive law of the soil establishes the stress-strain relationship, and can be written by:

$$\{\sigma\} = D^{ep}\{\varepsilon\} = D^{ep} \cdot S \cdot \{u\} \tag{7}$$

where σ denotes the stress, and ε is the strain. The matrix D^{ep} contains the constitutive behaviour of the soil.
Since, in the present study, it is expected that the fluid displacements are much higher than the solid phase movements, the solid skeleton is considered rigid, which means that *u=0* at all points, and subsequently, its first and second derivatives in time (velocity and acceleration). After applying this simplification to the mathematical model, the Eqs.(1), (2) and (3) finally are expressed by means of Eq.(8):

$$Q \cdot \nabla(\nabla^T\{w\}) - K^{-1}\{\dot{w}\} - \frac{\rho_w}{n}\{\ddot{w}\} + \rho_w\{b\} = \{0\}$$

(8)

Hence, the only remaining degrees of freedom at each point are the fluid phase displacement, *w*, along with its first and second derivatives in time.

2.2. Numerical tools

Eq.(8) is solved in the space domain by means of a Finite Element Method scheme, applying Galerkin's Method of Weighted Residuals (Ottosen & Petersson, 1992; Zienkiewicz and Taylor, 2000). A mesh composed by quadratic, triangular elements are used. Thus, the weak formulation of Eq.(8) is given by Eq.(9):

$$[K] \cdot \{w\} + [C] \cdot \{\dot{w}\} + [M] \cdot \{\ddot{w}\} = \{f_{ext}\}$$

(9)

where $\{\boldsymbol{w}\}, \{\dot{\boldsymbol{w}}\}, \{\ddot{\boldsymbol{w}}\}$ represent, respectively, the vectors of displacements, velocities and accelerations of the fluid phase in the space domain at both directions (x, y), and $\{\boldsymbol{f_{ext}}\}$ contains the vector of external nodal forces, including hydrostatic boundary conditions due to the water level outside the porous media, as well as the gravity forces. **[*K*]**, **[*C*]** and **[*M*]** represent the stiffness, damping and mass matrices.

The Newmark's step-by-step time integration scheme is employed in the time domain (Zienkiewicz and Taylor, 2000). An autoadaptative time integration scheme has been implemented, aiming to adapt the size of the time step to a limited numerical error in the approximation (Fernández-Merodo 2001).

2.2 Boundary conditions. Iterative procedure

The presented model computed the free surface and total amount of discharge through porous media under steady conditions, meaning that both upstream and downstream water levels outside the domain remain constant. In order to do that, two types of boundary conditions must be applied:

- Nodal forces at the upstream and downstream nodes, due to the hydrostatic pressures due to the external water levels: they are established at the beginning of the computation, and constant in a steady problem;
- Impermeability boundary conditions: these are not constant, but they must be changed following an iterative procedure. In fig. 1, this procedure is summarized. It shows the Muskat problem (Plaxis, 2010), consisting of a rectangular, homogeneous earth dam, 3.22 m high, and 1.62 m wide, the water levels being 3.22 m upstream (on the left) and 0.48 m downstream (on the right). In the first iteration, all the boundaries where there is no water outside the dam are let free, which means that the external imposed hydrostatic pressure is null at these locations, and then, fluid is free to pass through them (both coming in or going outside the geometry). If, after the end of the computation in this iteration (after reaching steady conditions), it is found that water comes into the soil through a boundary where it is not possible to happen, because there is no water outside

(unrealistic scenario), this border is taken as impervious for the next iteration to prevent this physically impossible situation (fig. 1a). It is not necessary to keep on iterating when, after two consecutive iterations, the boundaries remain unchanged. In figure 1b, the free surfaces obtained for the above mentioned Muskat problem after each iteration are sketched. Five iterations are needed in this case for completing the whole computation. This iterative procedure represents the main innovation of this displacement based approach to the unconfined seepage problem, since the space domain is constant throughout the whole calculation process, and the final solution is obtained after very few iterations (typically 4 or 5 in the most complicated cases, those in which the free surface intercepts the downstream boundary above the outside water level) (López-Querol et al., 2011).

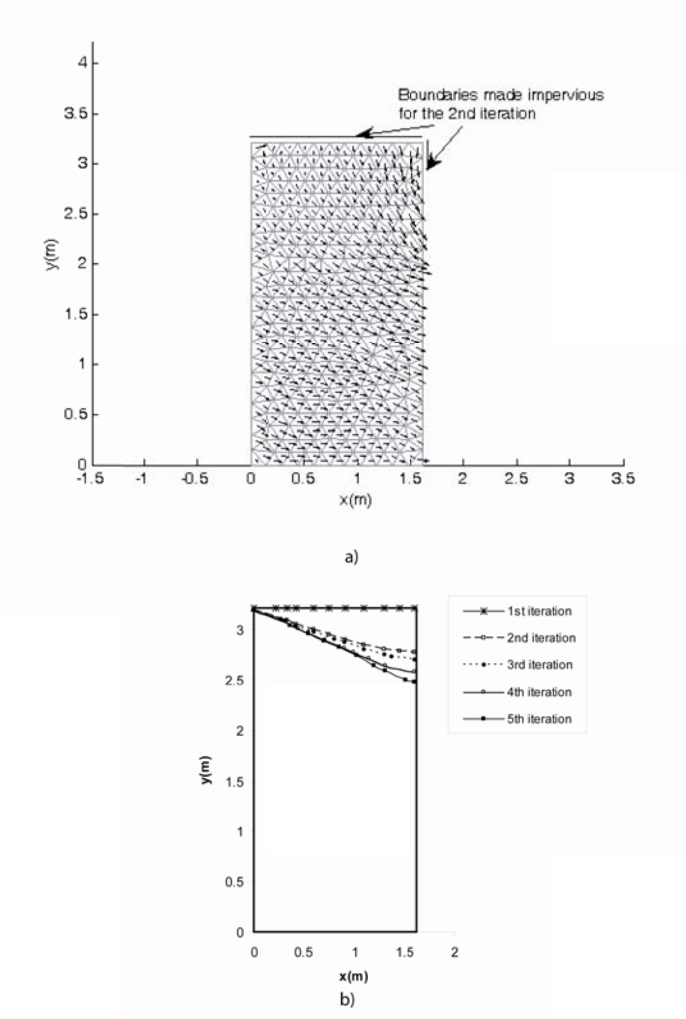


Fig. 1. Iterative procedure employed for computing the free surface in the Muskat problem, changing the impermeability boundary conditions. a) Water velocity vectors after the first iteration, and impermeability boundary conditions for the second iteration. b) Computed free surface after every iteration.

3. VALIDATION OF THE MODEL

Aiming to ascertain the accuracy and applicability of this new methodology, it has been validated by means of several theoretical cases with analytical solutions. For instance, Fig. 2 shows the geometry, up and downstream steady water levels, and pore pressure boundary conditions of a homogeneous earth dam with a toe drain, as well as the comparison of the location of the free surface obtained with the present model and analytical solutions obtained with Dupuit's and Numerov's methods, along with the numerical solution proposed by the Corps of Civil Engineers (Lambe and Whitman, 1979). The suitability of this methodology is self evident by the inspection of this figure.

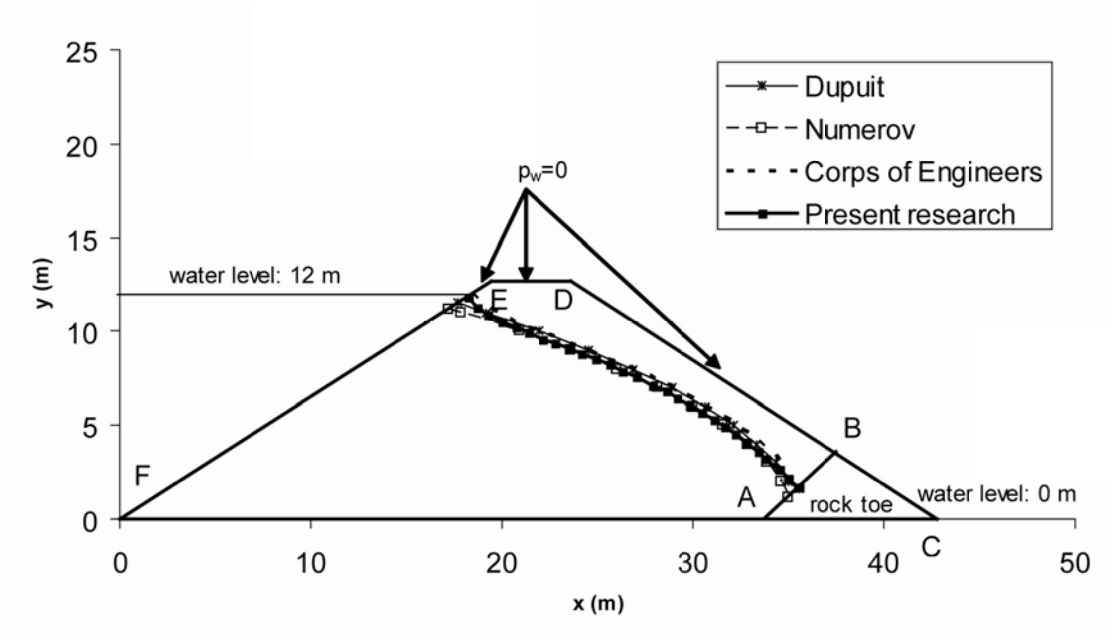


Fig. 2. Theoretical example of earth dam with available analytical solutions: homogeneous earth dam with a toe drain (Lambe and Whitman, 1979)

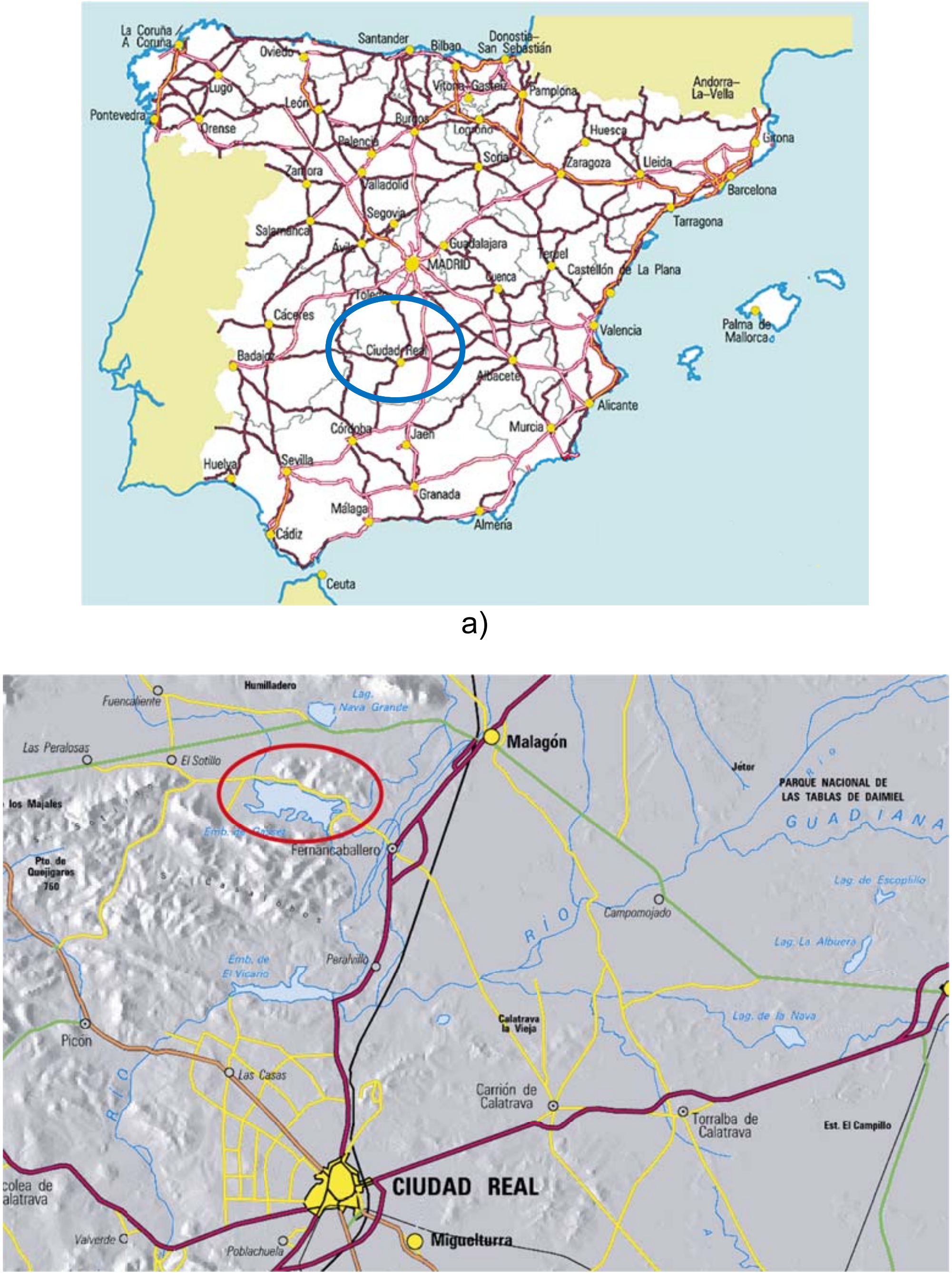
4. APPLICATION TO A REAL FIELD CASE: THE GASSET DAM (SPAIN)

4.1 Main features of the Gasset dam

The Gasset Dam was built in 1910, following the construction project developed by the Spanish engineer Mr. Bernando Granda in 1901. The dam is located over the Becea River in the village of Fernancaballero, located at the North of Ciudad Real, at the elevation of 611.40 meters above the sea level (elevation of the dam foundation). The Becea River begins at the South of Toledo, close to Porzuna, and finishes at the Bañuelos River three kilometers downstream of the Gasset dam (Fig. 3).

At the beginning, this dam consisted of a 13.5m tall, clayey – sandy core earth fill, upstream protected by a masonry slope with stairs of 1m and a global slope 1,7H:1V (Fig. 4a). The downstream side was a non continuum slope of 1,5H:1V until the upper shoulder, 2H:1V until the second one, and finally 2,3H:1V until the base of the dam.

The initial volume of the reservoir was 22,1Hm3 and the materials used for its construction were collected from the reservoir, mainly consisting of clay and quartzite cobbles.

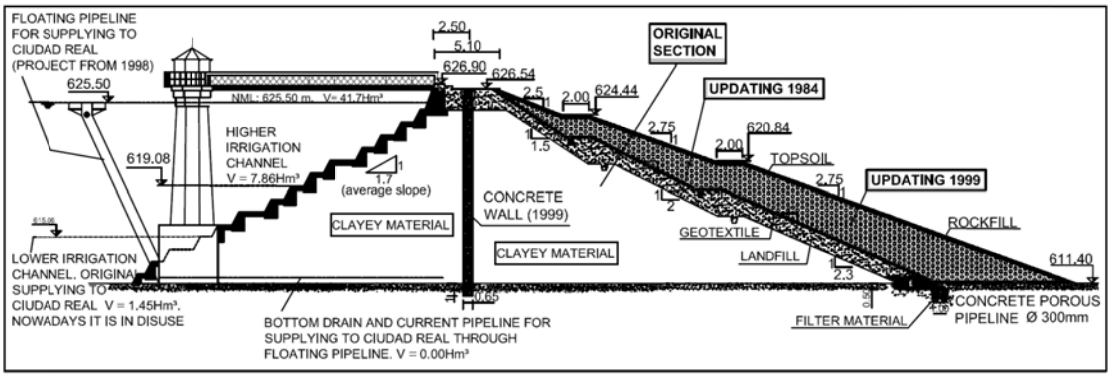


a)

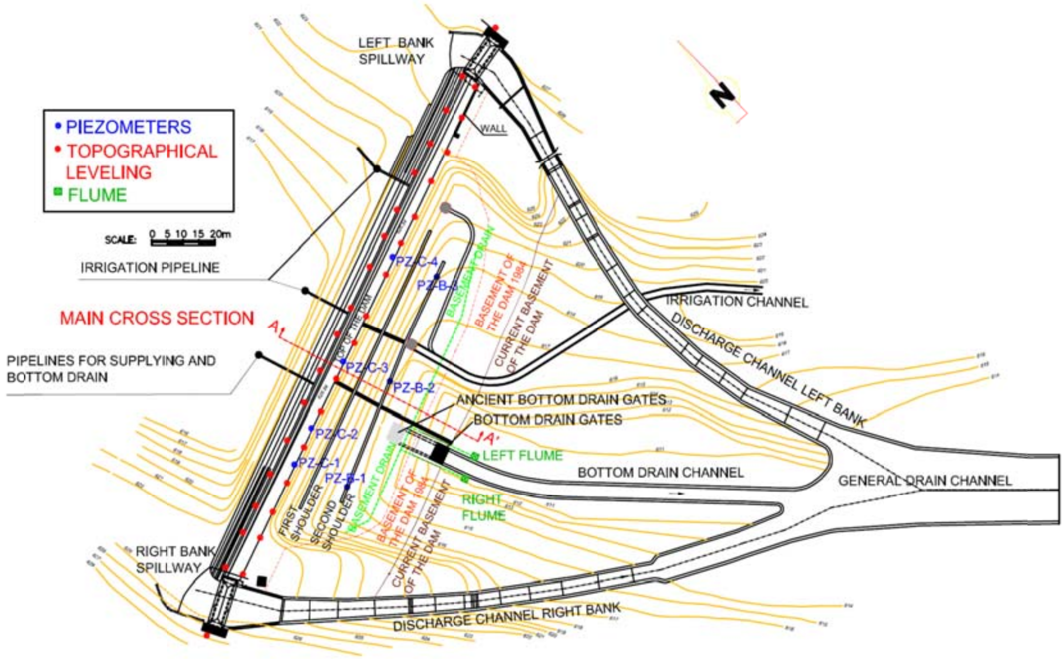
b)

Figure 3. Location map. a) Province of Ciudad Real. b) Gasset reservoir

In 1984, a new project was developed to increase the Gasset reservoir capacity (Fig. 4). The elevation of the dam was raised 1,5m. After that, the Gasset dam had a maximum total height of more than 15m, 5m wide at its top. With this works, the reservoir volume also increased up to its current maximum capacity to 41,7Hm3. During the same upgrading works, a group of 1 m thick drains at the downstream slope were built, as well as connected to a porous concrete pipeline located under the downstream foot of the dam. This pipeline collects all the filtrations through the dam and allows us to record the total amount of discharge through a Thompson flume of 90º.



a)



b)

Figure 4. Gasset dam. a) Historical main cross sections. b) Plan view.

In 1999, some downstream filtrations were detected, and hence, new works, basically consisting of the construction of a new thin concrete wall located in the central part of the dam (founded 1 m below the lower level of the dam) and a downstream rockfill slope over a geotextile, were undertaken. After that, the downstream slope became 2,5H:1V from the top of the dam until the upper shoulder (located 13m over the basement of the dam) and 2,75H:1V in the lower part. Over this new downstream slope, a layer of vegetal earth was extended to finish the works and this is the final configuration we can see nowadays. In summary, the whole geostructure is a heterogeneous earth fill dam, protected by masonry at the upstream slope and by rockfill in the downstream slope with two shoulders, including and impervious concrete wall inside (fig. 4a).

4.2 Application of the numerical procedure

The central cross section presented in fig.4a has been modelled using the mesh shown in Fig.5. This cross section is the one with maximum surface, and therefore it is the one where more filtration problems might take place, because of the maximum hydraulic gradient. The mesh was refined to accurately represent the different materials and features inside the geometry, as well as aiming the capture the higher hydraulic gradients at those locations where change of materials take place.

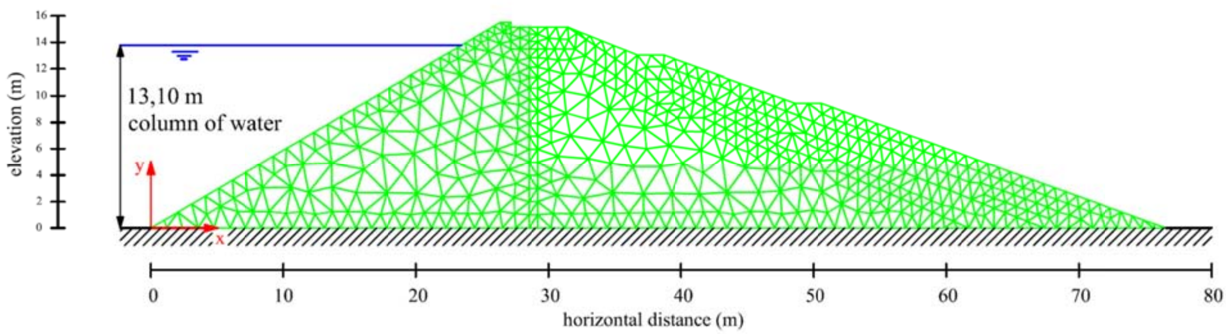


Fig. 5. Finite elements mesh used to discretize the domain of the type cross section in the Gasset dam.

In the computations, the water level at the reservoir was taken as 624.50 meters above the sea level, i.e. a water head of 13.10 meters referred to the basement of the dam, which more or less represents its maximum possible level. This steady water elevation was selected for computing purposes since it was the one for which field measurements were available. Several visits to the dam were made to collect these data of discharges and piezometric levels for this water level during the first months of 2010.

The geotechnical parameters used in the model were taken or computed from EPTISA (2001) and RODIO (1999). These values are: voids ratio, e = 0.39; porosity, n = 0.28; permeability, estimated through Hazen's formula (Lambe and Whitman, 1979): $K_1 = 10^{-6}$ m/s for the earth fill, $K_2=10^{-2}$m/s for the downstream rock fill, and $K_3 = 10^{-10}$ m/s for the concrete wall; compressibility of the solid particles, $K_s = 10^{40}$Pa; compressibility of the water: $K_w = 10^{9}$Pa; compressibility of the saturated soil (Zienkiewicz et al., 2000),

$Q=1.39 \cdot 10^{41}$ Pa. The rock foundation has been considered completely impervious, since it is a strong rock without generalized.

4.3.Field measurements

A visit to the dam to collect measurements in the piezometers was made on 18th March 2010. The total amount of discharge (filtrations) was also recorded in the flumes installed downstream. It is worth to point out that, as it has been already mentioned, the reservoir level was at 624,50 meter over the sea level. This elevation remained constant since the previous two months, and thus, steady conditions can be assumed. A second visit to the dam was made on 8th July 2010, when new measurements were collected. The reservoir level had slightly decreased until 624,37 meters over sea level, and it was addressed that the measures in the piezometers and flumes had not suffered significant changes (only variations from 2 to 5 cm in the piezometers and 2 mm in the flumes, corresponding to a total variation of discharge of 30 l/min, approximately, in a total discharge of 919.5 l/min). This second visit allowed us to corroborate the steady conditions assumed for the previous one. A summary of these collected measurements are given in Tables 1 and 2, where values of real water levels inside the dam at piezometers PZ-C-3 and PZ-B-2 (according to Figs. 4b and 6) and total amount of discharges due to filtrations are reported.

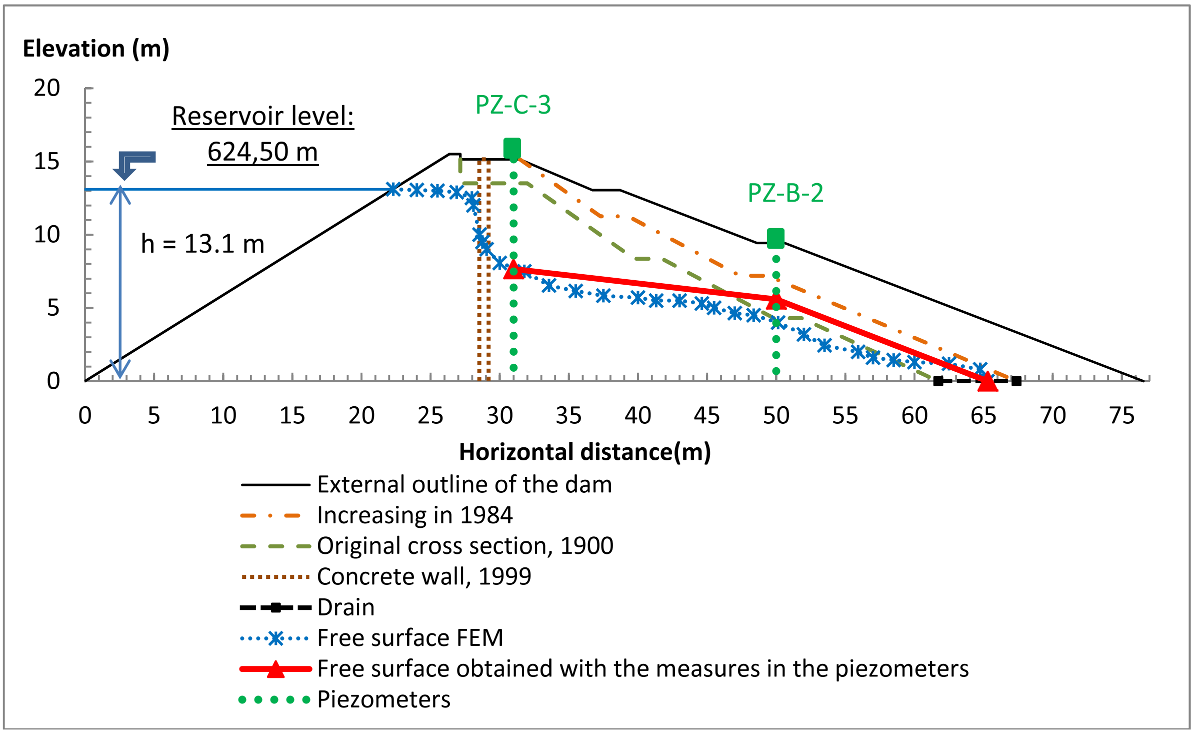


Fig. 6. Free surface position under steady conditions in the Gasset dam. Comparison between the real field piezometric levels and the numerical results obtained in the present research (free surface FEM).

Table 1. Measurements in the piezometers of the Gasset dam (date or measurements: 8th March 2010). (Bold and Italics indicate those piezometers located at the main cross section, used for comparison purposes at Fig. 6)

Location of the piezometer	Piezometer code	Measurement (m)	Elevation of the base of the dam (m)	Elevation of the column of water (m)	Elevation of the free surface from the base in the piezometer (m)
Top of the dam	PZ-C-1	8.10	614.50	10.00	3.94
	PZ-C-2	7.73	612.00	12.50	6.81
	PZ-C-3	***7.48***	***611.40***	***13.10***	***7.66***
	PZ-C-4	7.98	614.00	10.50	4.56
2nd shoulder	PZ-B-1	6.12	614.50	10.00	0.22
	PZ-B-2	***3.85***	***611.40***	***13.10***	***5.59***
	PZ-B-3	4.66	614.00	10.50	2.18

Table 2. Measurements in the flumes of the Gasset dam (date of measurements: 18th March 2010).

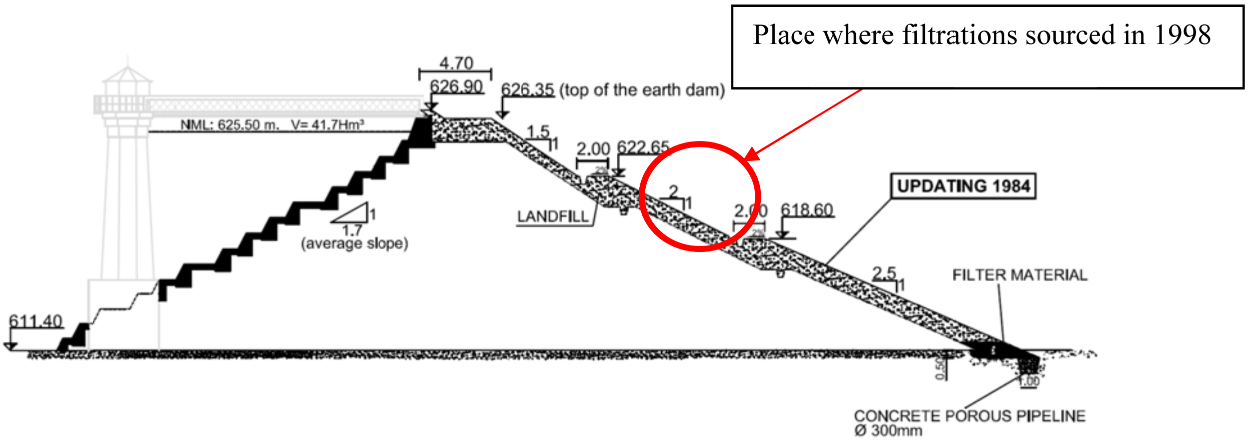
Location of the flume	Lecture (mm)	Discharge (l/min)
Right bank.	122	436.69
Left bank	127	482.82
	Total discharge:	**919.51**

4.4.Numerical results

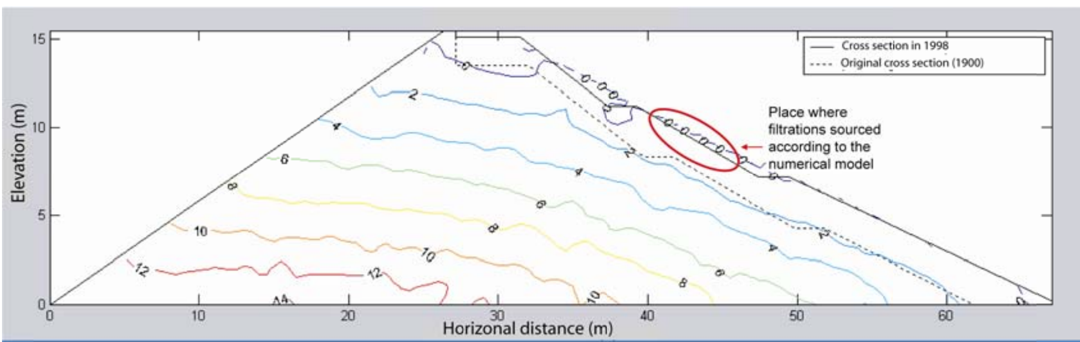
In Fig.6 the computed free surface location inside the earth fill dam under steady conditions is sketched. The comparison of the numerical solution with the piezometric field measurements is also given in the same figure. It is remarkable the approximation obtained with the numerical computation. The small differences between the real field measurements and the numerical results could be due to several reasons, like cracks inside producing unexpected filtration networks to occur, or uncertainties and variability of the soil properties maybe because the earth dam is not as homogeneous as assumed (the dam was built in 1900). The approximation given by the numerical model is good enough to confirm that the developed numerical code of finite elements is reliable. In the piezometer located by the impervious wall, the computation yields a level 32 cm below the measurement, while the result given by the model at the downstream piezometer is 1.09 m higher than the real value. This two calculations fit fairly well to the field data, given a good approximation of the free surface, as is self evident just having a look to Fig. 6.

The total amount of discharge through the earth dam under steady flow conditions, collected at the downstream drain, has also been determined with the developed numerical code, integrating the discharges at the elements close to the drain. Thus, the total amount of discharge collected at the downstream drain is q = *9.208 l/min/m*. The length of the drain built during the works conducted in 1984 is about 100 meters (Fig. 4b). Therefore, multiplying the computed q by the length of the drain, the total discharge collected in the drains of our numerical model of the Gasset dam gets 920.8 l/minute. The field measurement of discharge for the considered steady reservoir level was 919.51 l/minute. Thus, numerical and real values are almost identical.

As it has already been mentioned, several changes were performed in the Gasset dam during the 1998 works, amongst other reasons, trying to fix some leaks appearing at the downstream surface. The above described model has been employed herein to evaluate the possible problems suffered by the earth dam before it was repaired, aiming to identify the nature of that pathology. In February of 1998 the filtrations were found at an elevation of 620,68 meters over the sea level, between the irrigation pipeline and supplying pipeline (Fig. 7a). The water level at the reservoir was at 625,35 meters (EPTISA, 2001).



a)



b)

Figure 7. Downstream filtrations in 1998. a) Main cross section and field observations. b) Numerical model results (the free surface is the line 0 – atmospheric pressure).

The maximum cross section of the dam at this stage is shown in fig. 7a. This section consists of the original earth dam (dated in 1900) and its modifications carried out in 1984. Thus, it does not include the rockfill slope or the concrete wall either, since these works were done in 1999 in order to avoid the filtrations and to introduce deeper the piezometer level in the dam. These filtrations indicated problems or leaking into the dam, or maybe, that the drain located at the base was not effective enough. According to the technical inspection carried out in 1998, the water was appearing in between first and second shoulders.

The geotechnical parameters employed herein were the same than the previously used for the complete dam. The water level in the reservoir corresponds to a water column of 13,85 meters above the basement of the dam (625,35 meters above the sea level). The computation yields a free surface position shown in fig. 7b. It can be realised from this result that, in this case, the level of the free surface touches the downstream boundary at the same location where water was actually found. Two sources of water can be identified from this computation:

- The first one, located between the top of the dam and the upper platform. It is one small source because we can see how it comes back to entry into the earth dam.
- The second one, located between the first and the second shoulder. This is the most important one, and can be identified as the one observed in 1998, which justified the works carried out in the dam in 1999.

CONCLUSION

A new methodology for computing free surfaces in unconfined seepage problems has been applied to a real field case. The main conclusions obtained are summarized next:

- The suitability of this new methodology for real, heterogeneous earth dams under steady conditions has been ascertained.
- The numerical model is also valid for obtaining the total amount of discharge due to filtrations through the dam with high accuracy (error of 2‰), in spite of the simplicity and numerical efficiency of the presented formulation.
- This tool can be helpful for aiming to understand pathologies inside the earth dam, and for optimizing the design of repair works.

In summary: it has been ascertained the validity of the present formulation for real field cases. This tool is suitable to be employed for both designing new earth dams and upgrading old geostructures.

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