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Influence of Spatial Variability on Consolidation of Unsaturated Soils

Yan Cheng [1], Lulu Zhang [2], Zibo Zuo [3], and Jianhua Wang [4]

[1] *Graduate Student, Department of Civil Engineering, Shanghai Jiaotong University, Shanghai, China.*
[2] *Associate Professor, Department of Civil Engineering, Shanghai Jiaotong University, Shanghai, China.*
[3] *Graduate Student, Department of Civil Engineering, Shanghai Jiaotong University, Shanghai, China.*
[4] *Professor, Department of Civil Engineering, Shanghai Jiaotong University, Shanghai, China.*

ABSTRACT

Reliable prediction of consolidation rates in soil deposits is difficult due to highly spatial variability of in situ soil properties. In this paper, a numerical model for coupled consolidation of unsaturated soils is established combined with the random field method to investigate the influence of spatial variability on coupled consolidation of unsaturated soils. A fast and accurate of random field generation method, the Fast Fourier Transform method, is adopted to generate realizations of random field, which are then mapped onto the finite-element mesh of the numerical model. Parametric studies are conducted to investigate the effects of the coefficient of variation and the correlation length of saturated permeability k_s on the dissipation of excess pore water pressure and variation of settlement. The COV of k_s is greater or the correlation length is smaller means the spatial variability of saturated permeability is more significant. It is found that when the COV of k_s is greater, the mean settlement of the unsaturated soil is smaller while the COV of settlement is larger. When the correlation distance of k_s decreases, the mean settlement of the unsaturated soil decreases while the COV of settlement increases. Comparing with the results for the effect of the COV of k_s, the effect of the correlation distance is less significant. It is also found that when the spatial variability is more significant, the mean value and standard deviation of the generated excess pore water pressure are greater. The results imply that when the spatial variability of unsaturated soils is greater, the dissipation of excess pore water pressure is slower and hence the settlement of the soil is smaller.

1. INTRODUCTION

There is a wide variety of engineering problems where the consolidation of soils is important. In these problems, soils can be both saturated and unsaturated. For example, the construction of earth fill dams, highways and etc usually use unsaturated compacted soils. Large portion of land surface are covered with residual soils that are unsaturated and the ground water table is at some depth. Therefore, it is necessary to

*Corresponding author.
1 Graduate Student, E-mail: chengyan029@sjtu.edu.cn
2 Associate Professor, lulu_zhang@sjtu.edu.cn
3 Graduate Student, E-mail: zuobo2008@sjtu.edu.cn
4 Professor, E-mail: wjh417@sjtu.edu.cn

include both saturated and unsaturated consolidation. Some researchers have conducted studies to investigate the coupled consolidation in unsaturated soils. Wong et al. (1998) illustrated the numerical solutions of coupled consolidation for an unsaturated soil column. Following the formulation developed by Dakashanamurthy et al. (1984). Conte (2004) presented a solution for coupled consolidation in unsaturated soils in which both the air phase and water phase are continuous. Kim (2000) presented a coupled numerical model for land deformation in partially saturated soils due to surface loading.

In the previous studies, the soil is assumed to be homogenous. However, in reality, soil properties vary spatially even within the same soil layer as a result of deposition and post-deposition processes. The effect of inherent spatial variability of soil properties on the performance of geotechnical works has received considerable attention in recent years. Tartakovsky et al. (2003) analyzed unsaturated flow for heterogeneous soils with spatially distributed uncertain hydraulic parameters using the Kirchhoff transform to derive the moment equations for pressure head. More recently, Huang et al. (2010) conducted probabilistic analysis of coupled soil consolidation using the coupled Biot consolidation theory combined with the random finite-element method. Griffiths et al. (2011) described a methodology in which parameters such as the soil strength, slope geometry and pore pressures, are generated using random field theory. They illustrated examples for stability of an infinite slope by comparing the first order method and the random field methods. It is found that first order methods (e.g. FOSM and FORM) gave very similar results but inevitably underestimated the probability of failure compared with the random field analyses.

In this study, the spatial variability of soil properties will be considered systematically using random field theory (Vanmarcke 1977; Fenton and Griffiths 2008) and will be shown to have a considerable influence on consolidation of saturated-unsaturated soils. The influences of coefficient of variation of saturated permeability and its correlation length on the soil consolidation will be investigated by parametric studies.

2. THEORY AND METHODOLOGY

2.1 Coupled flow and deformation of unsaturated soils

The governing equations for groundwater flow in deforming unsaturated soils may be written as follows (Kim 2000):

$$\nabla(\mathbf{u}) + \left(n\frac{\mathrm{d}S_w}{\mathrm{d}h} + nS_w\beta_w\gamma_w \right)\frac{\partial h}{\partial t} + \alpha_c S_w \frac{\partial}{\partial t}\left(\frac{\partial u_k}{\partial x_k}\right) = 0 \tag{1}$$

$$\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\left(\frac{\partial u_k}{\partial x_k}\right)\delta_{ij} - (\alpha_c S_w \gamma_w h)^e \delta_{ij} \right] + \left[nS_w\rho_w + (1-n)\rho_s \right]^e g_i = 0, \quad i,j = x,y,z \tag{2}$$

where **u** denotes the water flow velocity tensor; h is the pore water pressure head; n is the porosity; S_w is the degree of water saturation ($0 \le S_w \le 1$); dS_w/dh is the specific water saturation capacity; γ_w is the unit weight of water (9.806×10^3 N/m^3); β_w is the compressibility of water (5.0×10^{-10} m^2/N); α_c is Biot's hydromechanical coupling coefficient or the effective stress coefficient ($0 \le \alpha_c \le 1$); u_k is the displacement in the k direction; x_k is the coordinate in the k direction; t is time; λ and μ are often referred to as Lamé's constants, with $\mu = G = E/2(1+v)$, $\lambda = Ev/(1+v)(1-2v)$, in which G is the shear modulus, E is the Young's modulus, and v is the Poisson's ratio; δ_{ij} is Kronecker's delta; ρ_s is the solid density; ρ_w is the density of water; g_i is the component of gravitational acceleration g in the i direction; the superscripts e denote the incremental values of physical quantities.

According to Darcy's law, the water flow velocity **u** can be written as:

$$\mathbf{u} = -\mathbf{k}\nabla H \tag{3}$$

where $\boldsymbol{k} = k_r \boldsymbol{k}_{sat}$ is the effective hydraulic conductivity tensor, k_r is the relative hydraulic conductivity ($0 \le k_r \le 1$), and $\boldsymbol{k}_{sat}$ is the saturated hydraulic conductivity tensor; $H = h+z$ is the (total) hydraulic head, z is the vertical axis and elevation head.

In a plane strain condition, the above governing equations, i.e., Eqs. (1) and (2), can be simplified as follows.

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial}{\partial x}(h+y)\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial}{\partial y}(h+y)\right) = \left(n\frac{\mathrm{d}S_w}{\mathrm{d}h} + nS_w\beta_w\gamma_w\right)\frac{\partial h}{\partial t} + \alpha_c S_w \frac{\partial}{\partial t}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] \tag{4}$$

$$\frac{\partial}{\partial x}\left[G\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right) + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y}\right) - (\alpha_c S_w \gamma_w h)^e\right] + \frac{\partial}{\partial y}\left[G\left(\frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x}\right)\right] = 0 \tag{5}$$

$$\frac{\partial}{\partial y}\left[G\left(\frac{\partial \upsilon}{\partial y} + \frac{\partial \upsilon}{\partial y}\right) + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y}\right) - (\alpha_c S_w \gamma_w h)^e\right] + \frac{\partial}{\partial y}\left[G\left(\frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x}\right)\right] + \left[nS_w\rho_w + (1-n)\rho_s\right]^e g = 0 \tag{6}$$

where k_x and k_y are the hydraulic conductivities in x and y direction, respectively; u, v denote the displacements in x and y direction, respectively. It should be noted that in an unsaturated soil, the pressure head h is negative and both the degree of saturation S_w and the relative hydraulic conductivity k_r are dependent upon the pressure head h.

In this study, a finite element model of the coupled governing equations [Eqs. (4), (5), and (6)] is developed in the commercial multiphysics modeling and simulation software COMSOL (COMSOL AB. 2008). An illustrative example of coupled consolidation in an saturated-unsaturated soil column is presented to illustrate the application of the numerical model.

2.2 Random field theory

The main source of soil heterogeneity is inherent spatial variability of soil. That means soil properties vary from one point to another in space due to different depositional condition and loading histories. The spatial variation of soil properties can be described using the random field theory (Vanmarcke 1977; DeGroot and Baecher 1993; Fenton and Griffiths 2008). The uncertainties in the spatial-averaged soil properties in a given domain with homogenous soils are influenced by the correlation structures of the soil properties at different locations. The correlation structure is often assumed to be a simple function of distance between points, governed by a single parameter. A common used model is one in which the correlation decays exponentially with the distance between points:

$$\rho(\tau) = \exp\left(-\frac{2\tau}{\delta}\right) \tag{7}$$

where τ is the distance between two points; δ is the scale of fluctuation or correlation length. The correlation length of soil δ is the important indicator of soil spatial autocorrelation related characteristics describing the distance over which the spatially random values will tend to be significantly correlated. That is to say, apart in a range of δ, soil index is relevant; but in a range over δ, soil index is irrelevant. Table1 lists some common correlation functions with their expressions and correlation lengths.

Table 1 Common correlation functions of random field

Correlation function	Expression	Correlation length
single exponential	$e^{-\tau/a}$	$\delta = 2a$
double exponential	$e^{-(\tau/a)^2}$	$\delta = \sqrt{\pi}b$
index cosine	$e^{-\tau/c}\cos(\tau/c)$	$\delta = c$
triangular	$1-\tau/h$; $\tau < h$ 0; $\tau > h$	$\delta = h$

2.3 Generation of random field samples

Different random field generator algorithms are available of which the following are the most common: Moving-Average (MA) method, Covariance Matrix Decomposition method (Michael 1987), Discrete Fourier Transform (DFT) method, Fast Fourier Transform (FFT) method (Cooley and Tukey 1965), Turning-Bands method (TBA) (Matheron 1973; Robin et al. 1997), Local Average Subdivision (LAS) method (Fenton 1990). In this paper, the Covariance Matrix Decomposition method is adopted to generate samples of random field.

The Covariance Matrix Decomposition method is a direct method to produce a homogeneous random field assuming the parameters at different locations in the field are correlated random variables. Assume the covariance matrix for these correlated random variables is **C**. Based on the random field theory, the prescribed covariance structure is $\mathbf{C}(x_i\text{-}x_j)=\mathbf{C}(\tau_{ij})$, where x_i, i=1,2,…,n, are discrete points in the field and τ_{ij} is the distance vector between spatial points x_i and x_j. If the covariance matrix **C** is positive definite, then mean zero correlated Gaussian random variables $Z_i=Z(x_i)$ can be produced using mean zero independent Gaussian random variables according to the following equation:

$$\mathbf{Z} = \mathbf{LU} \tag{8}$$

where **L** is a lower triangular matrix satisfying $\mathbf{LL}^T = \mathbf{C}$; **U** is a vector of n-independent mean zero, unit-variance Gaussian random variables; **Z** is a vector of n-correlated mean zero, unit-variance Gaussian random variables. The lower triangular matrix **L** is typically obtained using the Cholesky decomposition method.

The Covariance Matrix Decomposition method is simple and accurate. It is usually used for small fields with small size of discrete points. With the increasing computation capacity in these days, the storage problem of large size of random field can be solved easily. However, considerable round-off error may be induced in Cholesky decomposition when covariance matrices are often poorly conditioned and become numerically singular.

3. AN EXAMPLE

3.1 Model description

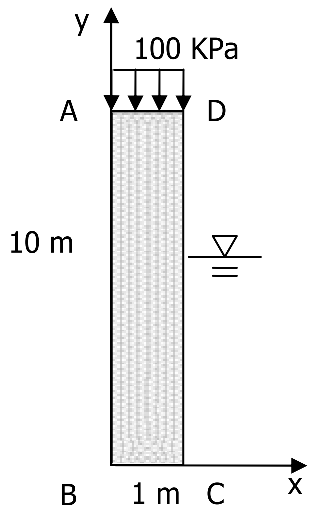

Fig. 1 An illustrative example of unsaturated soil consolidation

As is shown in Fig. 1, a hypothetical 10-m-high sandy soil column is used as an illustrative example. The initial ground water table is located at 6m above the bottom surface of the column. The soil above the water table is assumed to be unsaturated with an initially hydrostatic pore pressure. A surcharge load is applied instantaneously on the ground surface with a magnitude of 100 Kpa. The soil then consolidates due to the surcharge load.

The boundary conditions for stress-deformation and water flow are defined as follows. For stress and deformation of the soil skeleton, AB and CD are fixed horizontally but can move vertically; the bottom surface BC is fixed both vertically and horizontally. A surcharge load of 100 kPa is applied on the top surface AD. For water flow, AB and CD are impermeable. The bottom surface BC is assumed to be impermeable. The top surface AD is a drained boundary which means no accumulation of water is allowed on top surface. The soil column is discredited into 226 triangular elements in the finite element program. The an average size of the element of 0.3 m.

3.2 Material properties

The soil has three most important properties of interest to the consolidation problem: the elastic modulus, the poison ratio and the permeability. In this study, as we focus on the effect of soil hydraulic properties, only the saturated permeability k_s is assumed to a spatial random soil property. Random fields of k_s are generated using the previous described Covariance Matrix Decomposition method. Parametric studies are performed to investigate the effect of the coefficient of variation and the correlation distance of saturated permeability k_s on the dissipation of excess pore water pressure and variation of settlement. Here, COV_k is used to describe the coefficient of variation of saturated permeability k_s while δ is the correlation length of saturated permeability k_s. It is assumed the correlation length along the vertical direction is equal to the correlation length along the horizontal direction in this study. It is however often found that the horizontal correlation length is much larger than the vertical one in situ, which is mainly due to the in-situ heterogeneity of soil properties. In this study, we focus on the dissipation of excess pore-water pressure and settlement of soil along the vertical direction. The deformation and water flow along the horizontal direction is negligible. Hence, the assumption of spatially isotropy is reasonable in this example. In the parametric study, COV_k is assumed to vary as 0.125, 0.2, 0.5, 1, 2 and 4 and δ is 1, 2, 5, 10 and 20 m.

The functions of the degree of saturation and the relative hydraulic conductivity of the unsaturated soil are defined as follows.

$$S_w = \begin{cases} S_{wr} + \dfrac{1 - S_{wr}}{\left\{1 + \left[\beta(-h)^{\gamma}\right]\right\}^{\alpha}} & h < 0 \\ 1 & h \geq 0 \end{cases} \tag{8}$$

$$k_r = \begin{cases} \left\{1 + \left[a(-h)^{b}\right]\right\}^{-\alpha} & h < 0 \\ 1 & h \geq 0 \end{cases} \tag{9}$$

where S_{wr} is the residual degree of saturation; and α, β, γ, a and b are the unsaturated hydraulic parameters. The values of all the deterministic soil parameters are summarized in Table 2.

Table 2 Input parameters of the numerical model.

Parameters (Unit)	*Value*	*Parameters (Unit)*	*Value*
ρ_w (kg/m^3)	1000	v	0.3
g (m/s^2)	9.8	S_{wr}	0.07
ρ_s (kg/m^3)	2.65×10^3	α	1.0
β_w (m^2/kN)	5×10^{-7}	β (m^{-1})	1.74
α_c	1.0	γ	2.5
n	0.45	a (m^{-1})	6.67
E (kPa)	1.9×10^4	b	5.00

4. RESULTS AND DISCUSSION

4.1 Consolidation of unsaturated soil

Fig. 2 shows the mean excess pore water pressure along y direction for the case with COV_k = 0.5 and δ = 0.1. As the ground surface is free of drainage, in the unsaturated zone the excess pore water pressure dissipates very fast and little excess pore water pressure exist. The generated excess pore water pressure due to surcharge load increases with depth as the bottom surface is impermeable. The excess pore water pressure gradually decreases as time increases under the water table. In addition, as time elapses, the pore-water below the original ground water table moves upwards to the unsaturated zone. Hence, the pore-water pressure in the unsaturated zone increases. After 1 days of consolidation, the excess pressure reaches to about 0.1 m in both saturated and unsaturated zones. It is also shown that COV of the excess pore pressure is greater than 50% initially and gradually reduces to 20%.

Fig. 3 illustrates the variation of ground settlement with respect to time. It can be seen that the initial settlement due to elastic deformation of the unsaturated-saturated soil column is 36.5 mm. After around 2 hours of consolidation, the settlement is increased to 38.1mm and become stable after that. The COV of the settlement is initially around 13.5% and slightly reduces to around 13%.

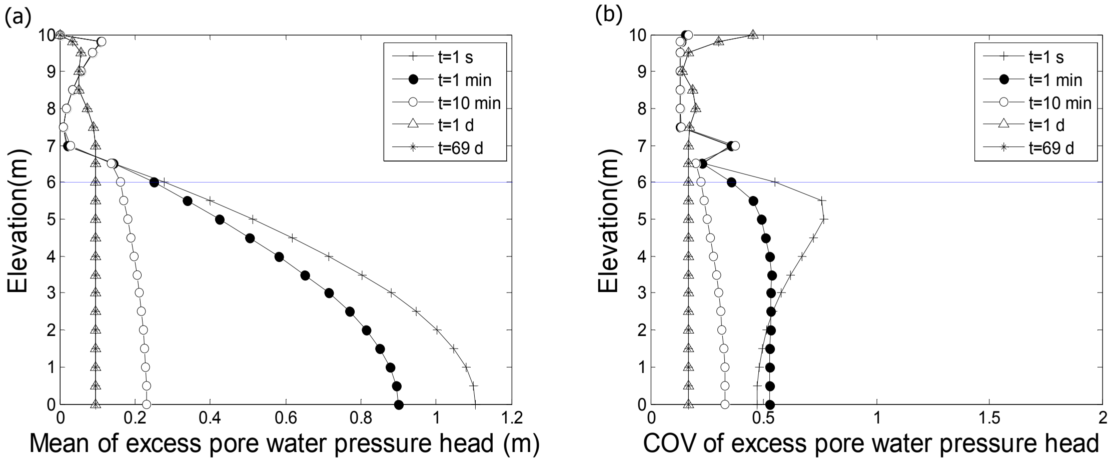

Fig. 2 Variation of excess pore-water pressure head with time
(COV_k = 0.5 and δ = 0.1)

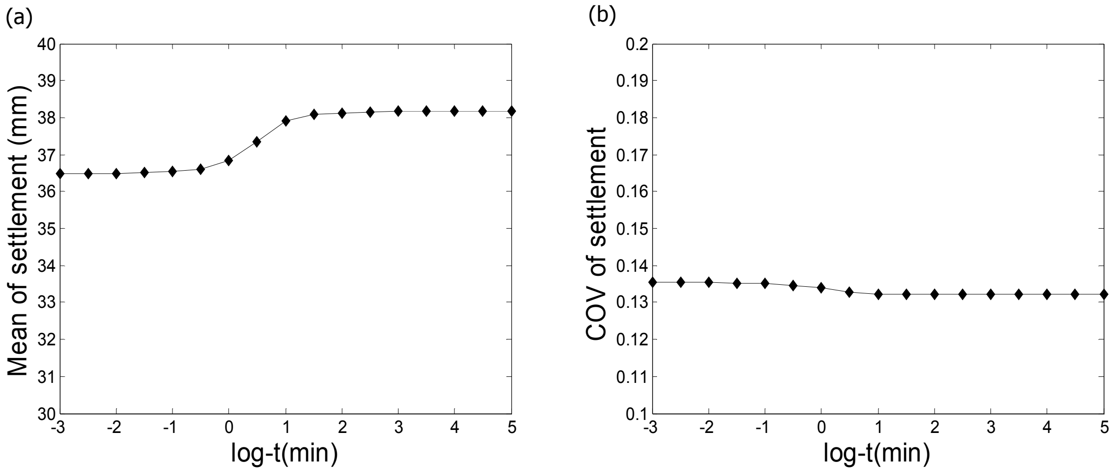

Fig. 3 Variation of ground surface settelement with time (COV_k = 0.5 and δ = 0.1):
(a) mean settlement, (b) COV of settlement

4.2 Effect of COV of saturated permeability

Fig. 4(a) shows the effect of the point COV of saturated permeability on the settlement of the soil column. The correlation length is assumed to be 0.1. It is found that when the COV of k_s is greater, the initial mean settlement of the soil is smaller. This is because as the point COV is larger, the heterogeneity of soil due to spatial variability is more significant. Hence, the less permeable zones in the soil are more frequently encountered and the water flow is slower. As a consequence, the consolidation of the soil is less significant in a soil with large spatial variability of permeability. As shown in Fig. 4(b), when the COV of k_s is greater, the COV of settlement is larger. As the consolidation proceeds, the COV of settlement is reduced slightly with time.

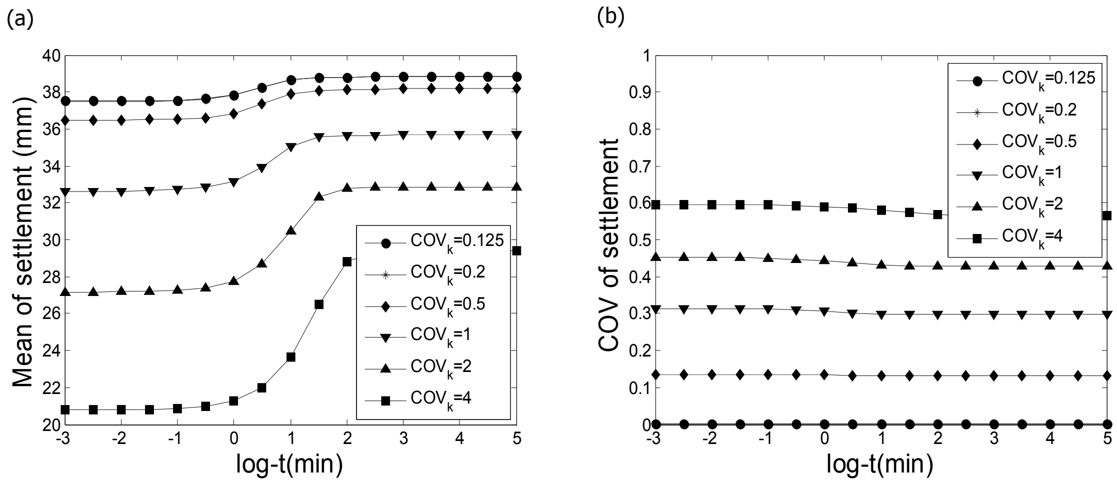

Fig. 4 Effect of COV$_k$ on the ground settlement
(a) mean settlement, (b) COV of settlement

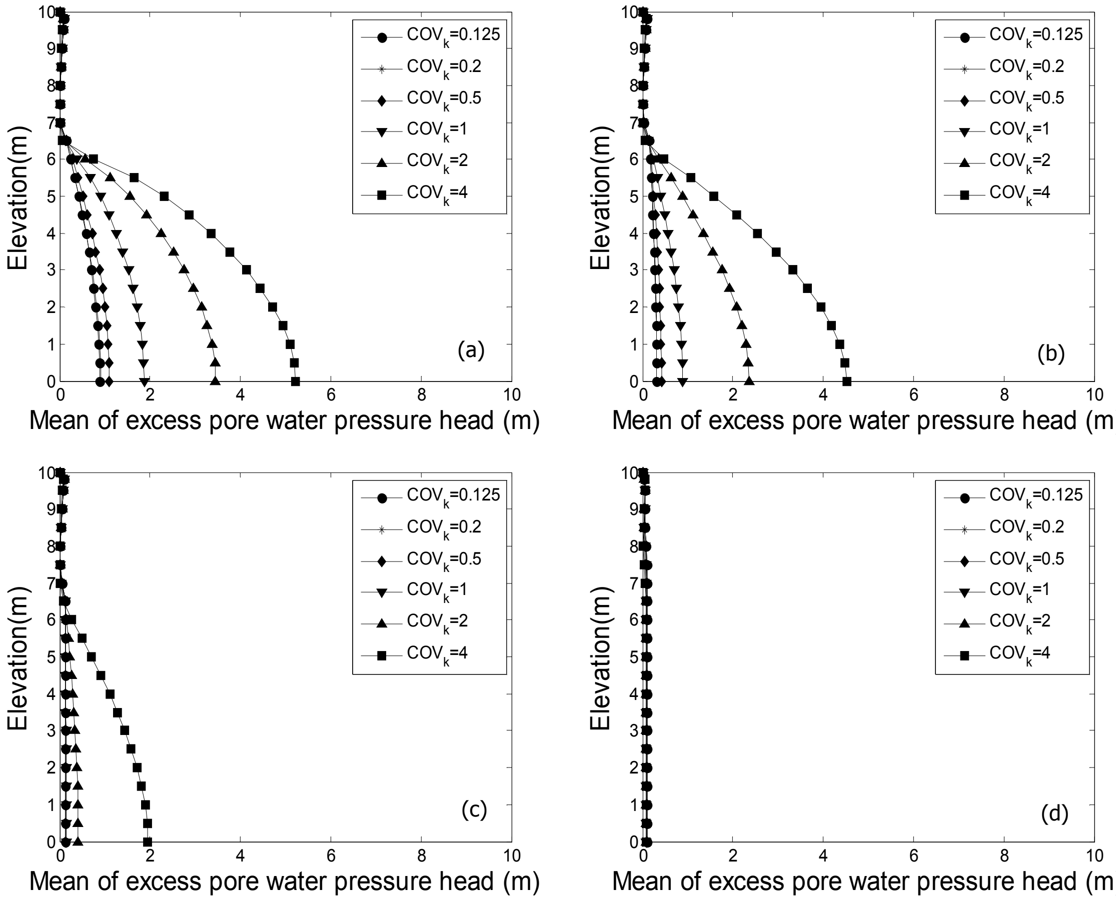

Fig. 5 Effect of COV$_k$ on mean pore water pressure head profiles:
(a) t = 1 s; (b) t = 5 min; (c) t = 30 min; (d) t = 1 d.

As shown in Fig. 5, significant difference of pore water pressure profiles can be observed for different COV of saturated permeability. With a greater value of COV$_k$, the dissipation of excess pore pressure is slower initially. However, after a day of consolidation, excess pore water pressure is dissipated to almost zero and there is not much difference for the soils with different COV_k. Fig. 6 shows the effect of COV_k on the uncertainty of excess pore pressure at 1s. It can be seen that when the spatial variability is more significant, the uncertainty of the generated excess pore water pressure are generally greater. In the soils around the ground water table, the COV of excess pore pressure is higher than the soils in the unsaturated zone

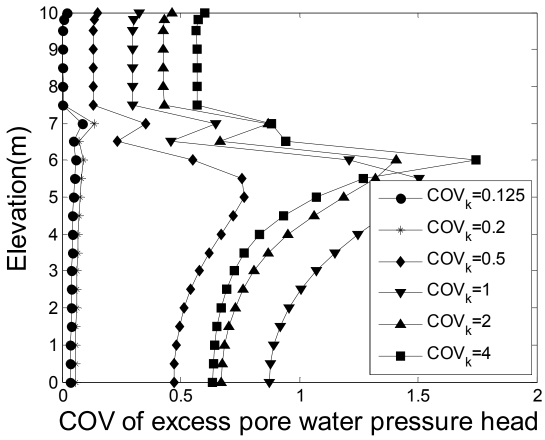

Fig. 6 Effect of COV_k on the COV of excess pore water pressure head (t = 1 s).

4.3 Effect of correlation length

The effect of the correlation length of saturated permeability on the mean and COV of settlement is illustrated in Fig. 7. In these cases, the value of COV_k is all 0.5. It is found that when δ is larger, the mean value of the settlement become greater while the COV of the settlement is smaller. A larger value of correlation length corresponds to a more uniform soil property field and hence the consolidation is faster.

Fig. 8 illustrates the variation of mean of excess pore water pressure with times for different values of correlation length. As is shown in the figure, when the correlation length is larger, the dissipation of the excess pore water pressure is faster. Fig. 9 shows the effect of the correlation length on the uncertainty of excess pore pressure at 1s. It can be seen that when the correlation length is larger and the spatial variability is less significant, the uncertainty of the generated excess pore water pressure are smaller. In the soils around and below the ground water table, the COV of excess pore pressure is higher than the soils in the unsaturated zone.

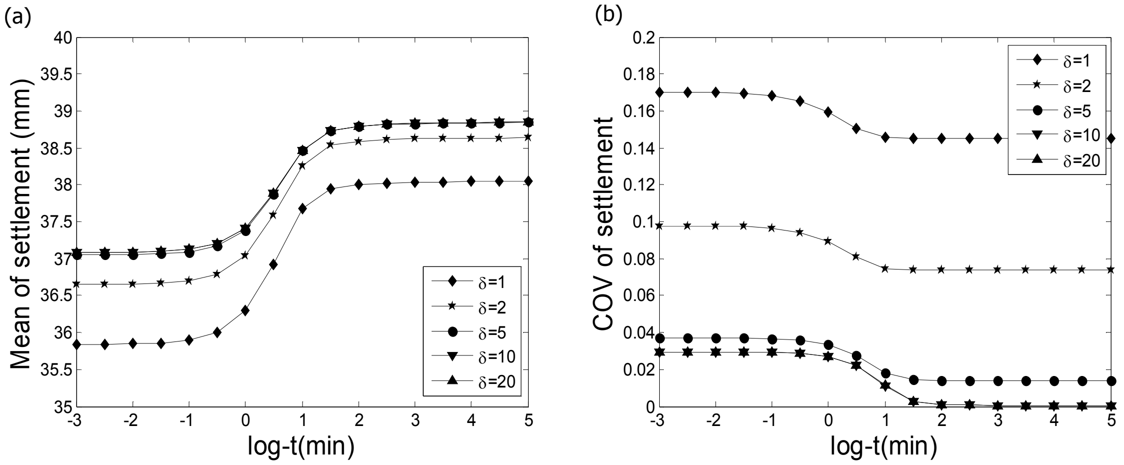

Fig. 7 Effect of correlation length on settlement:
(a) mean settlement; (b) COV of settlement

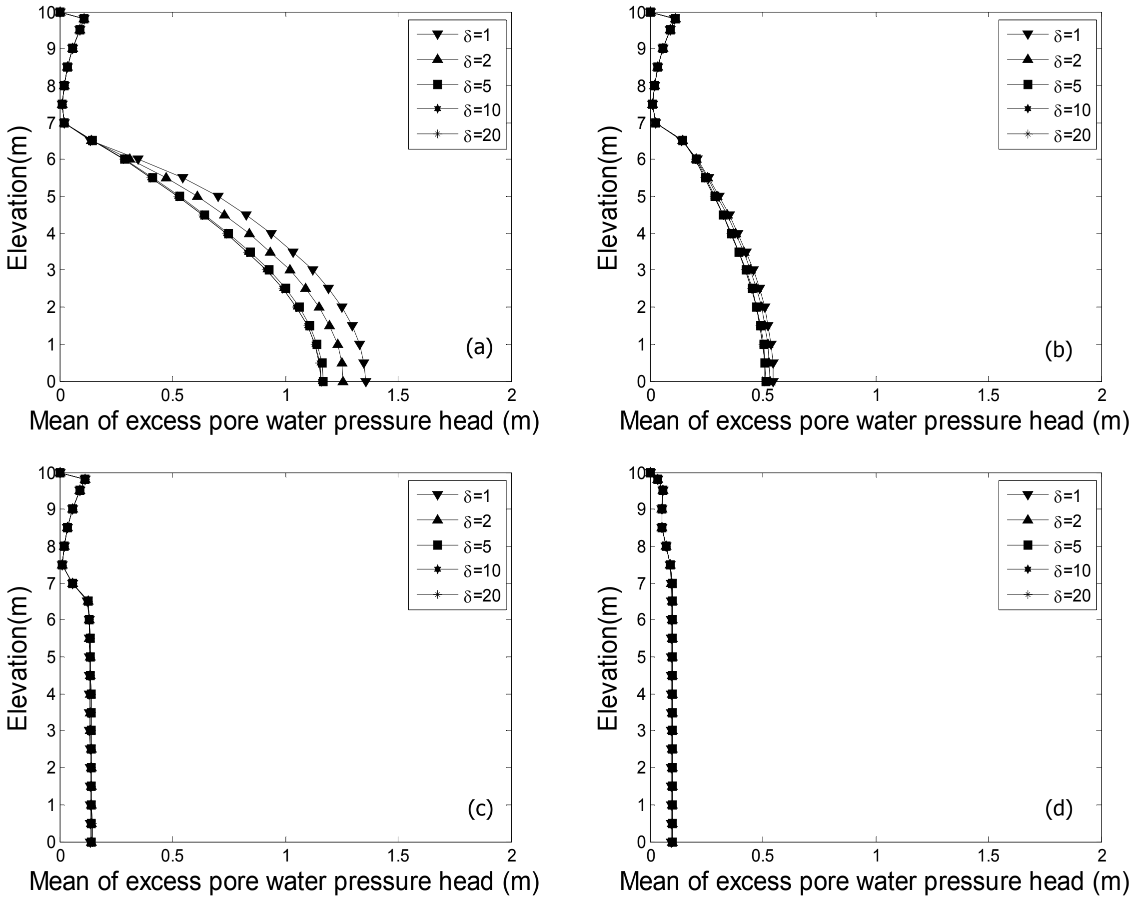

Fig. 8 Influence of correlation length on dissipation of excess pore water pressure head:
(a) t = 1s; (b) t = 5 min; (c) t = 30 min; (d) t = 1 d.

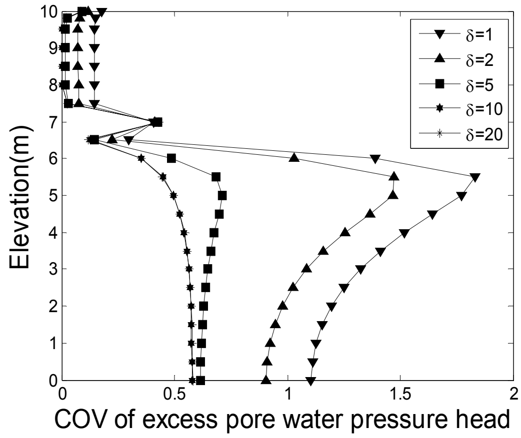

Fig. 9 Effect of correlation length on the COV of excess pore water pressure head (t = 1 s).

CONCLUSION

In this paper, a numerical model for coupled consolidation of unsaturated soils is established combined with the random field method to investigate the influence of spatial variability on coupled consolidation of unsaturated soils. The major findings are as follows:

1) With the same value of the correlation length, when the COV of k_s is greater, the dissipation of excess pore pressure is slower which results in a smaller mean settlement of the soil. With a larger COV_k, the COV of settlement is larger.
2) When δ is larger, the mean value of the settlement become greater while the COV of the settlement is smaller.

This paper studied the settlement and dissipation of excess pore water pressure head analyzing the only property of the saturated permeability k_s without considering the spatial variability of elastic modulus and Poisson ratio. Effects of spatial variability of both soil deformation properties and hydraulic properties will be investigated in future research study.

Acknowledgments
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