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Engineered Soils: Thermal Conductivity

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ABSTRACT

Thermal conductivity has a profound impact on the design and performance of energy-related geosystems. In some cases, high thermal conductivity is advantageous such as for the foundation of nuclear reactors, ground heat pump exchangers and the production of geothermal reservoirs. However, low thermal conduction may also be sought for example for the foundation and isolation of molten salt thermal storage systems. Heat transfer in soils involves diverse conduction mechanisms, starting with lattice vibrations in non-metallic mineral particles, the movement of electrons in metallic components, and molecular collisions and convection in fluids. As these phases come together to form the particulate medium, thermal conduction takes place through particle-particle, particle-fluid, and particle-fluid-particle heat conduction paths. Multiple conduction mechanisms and paths, pore fluids and grain mineralogy give rise to a broad thermal conductivity range in natural geomaterials (typically between 10^{-2} to 10^{0} $W \cdot m^{-1} \cdot K^{-1}$), and open the opportunity to engineer soils to attain unique thermal characteristics. Experimental and analytical results evaluate the performance of granular mixtures and show the controlling effect of the pore fluid.

1. THERMAL CONDUCTIVITY IN GRANULAR MEDIA

Granular media are composed of solid, liquid and gas phases, each of which possesses distinct thermophysical properties. Heat transfer by conduction differs radically among phases both in nature and magnitude.

1.1 Thermal conductivity in solid phases

Heat conduction in amorphous and crystalline dielectric solids occurs through lattice vibration waves, i.e., phonons. Thermal conductivity in non-metallic minerals is a function of temperature; in terms of the Debye temperature θ corresponding to the highest phonon vibration mode: (1) at low temperatures $T<⅓\theta$, atom vibrations are highly correlated and can be described by the propagation of elastic waves in a continuum, (2) at intermediate temperatures $⅓\theta < T < \theta$, the lattice vibration mode leads to dispersion, (3) at high temperatures $T \rightarrow \theta$, atoms vibrate independently of their neighbors (Touloukian et al. 1970).

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Thermal conduction in metals takes place through lattice vibration waves and freely moving valence electrons. Phonon heat conduction in metals, alloys, and semimetals is of the same order of magnitude as that of non-metals of similar elastic properties. Electron heat conduction parallels electrical conductivity by the Wiedemann-Franz law (Touloukian et al. 1970).

Heat-induced deformation in amorphous or semi crystalline polymers changes the internal structure, deters heat transfer, and renders polymers in general poor thermal conductors. At ambient temperatures the thermal conductivities of most plastics range between 5×10^{-2} and 10^{0} $W\cdot m^{-1}\cdot K^{-1}$ (Bhargava 2004). The introduction of thermally conductive fillers yields a linear increase in density and heat capacity and can produce an exponential increase in thermal conductivity and diffusivity depending on the degree of interconnectivity of the filler (Chung 2001; Droval et al. 2006).

1.2 Thermal conductivity in fluid phases

Bridgman's equation captures the key variables that determine the thermal conductivity λ_T in liquids (Bridgman 1923):

$$\lambda_T = 2\alpha v \delta^{-2} \tag{1}$$

where α is the Boltzman-Drude constant (2.02×10^{-16}), v is the speed of sound in the liquid, and δ the mean separation distance between the molecules in a simple cubical array. Bridgman's equation can be written as (Bird et al. 1960):

$$\lambda_T = 2.8 \cdot \kappa \cdot v \cdot \left(\frac{N}{V}\right)^{2/3} \tag{2}$$

where κ is the Boltzmann constant (1.380×10^{-23} $j\cdot K^{-1}$), the factor V/N is the volume per molecule, and $(V/N)^{1/3}$ is the center-to-center distance in a simple cubic packing configuration.

In the case of gases, the mean-free path theory provides an estimate of the thermal conductivity based on energy transfer though inter-molecular collisions. For dilute gases, (Touloukian et al. 1970):

$$\lambda_T = \frac{1}{3} \cdot \rho \cdot \tilde{v} \cdot \delta \cdot C_v \tag{3}$$

where ρ is the gas density, $\tilde{v}$ is the mean molecular speed, δ is the mean free path, and C_v is the heat capacity of the gas at constant volume.

1.3 Thermal conductivity in dry granular media

The thermal conductivity of mono-size lead granular beds in vacuum is 0.07 $W\cdot m^{-1}\cdot K^{-1}$ (Masamune and Smith 1963), which corresponds to 0.2% of the thermal conductivity of solid lead, i.e., 35.3 $W\cdot m^{-1}\cdot K^{-1}$. Similarly, the thermal conductivity of a packing of mono-size glass particles in vacuum is 0.05 $W\cdot m^{-1}\cdot K^{-1}$ which corresponds to 5% of the thermal conductivity of solid glass (1.05 $W\cdot m^{-1}\cdot K^{-1}$). Furthermore, the thermal conductivity of glass particle packings in vacuum is independent of particle size; however, this is not the case when air is present in the pore space: in air, the effective thermal conductivity of the packing increases with gas pressure at a rate that is particle-size-dependent (Masamune and Smith 1963).

Tests on mixtures of binary-size spherical glass particles with a size ratio R_d = 2.6 show that the effective thermal conductivity traces porosity i.e., interparticle coordination: the effective thermal conductivity peaks when porosity is at a minimum (Guyon et al. 1987; Okazaki et al. 1981).

The thermal conductivity of mono-size mixtures of copper (400 $W \cdot m^{-1} \cdot K^{-1}$) and glass (1.05 $W \cdot m^{-1} \cdot K^{-1}$) spherical particles increases linearly with the fraction of copper. An increase in particle size results in a uniform increase in the effective thermal conductivity for all glass-copper mixtures (Okazaki et al. 1981).

1.4 Thermal conductivity of saturated granular media

The effective thermal conductivity of granular media is strongly affected by the presence of liquids. Experiments results for water-saturated glass particle packings show an increase in thermal conductivity from 0.153 $W \cdot m^{-1} \cdot K^{-1}$ for air-filled specimens to 0.797 $W \cdot m^{-1} \cdot K^{-1}$ for water saturated specimens (Gori et al. 2001).

1.5 Summary

These results highlight the importance of contacts on thermal conductivity in dry sediments, and the contribution of fluid conduction along alternative conduction paths next to contacts (Yun and Santamarina 2008). Indeed, experimental dataset show that thermal conductivity in granular media is a function of the mineralogical composition of the solid phases (Gangadhara Rao and Singh 1999; Tarnawski et al. 2002), particle size (Aduda 1996; Batchelor and O'Brien 1977; Gangadhara Rao and Singh 1999; Tarnawski et al. 2002), packing density and coordination number (Tarnawski et al. 2002), water content (Andersland and Ladanyi 2004; Farouki 1985; Lu et al. 2007; Singh and Devid 2000), and effective stress (Batchelor and O'Brien 1977; Chung 2001; Cortes et al. 2009; Droval et al. 2006; Vargas and McCarthy 2001; Yagi and Kunii 1957).

2. EXPERIMENTAL STUDY

We explore herein the effects of high thermal conductivity solid metallic intrusions in coarse grained soils, and the effects of changes in pore fluid in the effective thermal conductivity of granular mixtures.

2.1 Measurement method and calibration

The thermal needle method is used in this study (Stalhane and Pyk 1931). The linear heat source is an electrical wire connected to a current source to create a constant heat input q per unit length and time. A thermocouple at the center of the needle tracks the temperature evolution with time. The thermal conductivity can be inverted from the temperature-time T-t record as:

$$\lambda_T = \frac{q}{4\pi\Delta T}\ln\left(\frac{t}{t_o}\right) \qquad (4)$$

where ΔT is the temperature change between time t_o and t.

The needle probe method was calibrated using gelated water specimens to prevent convective transport. Calibrations using different specimen size and chamber materials show no boundary effects on measurement conducted in chambers as small as 15 mm in diameter when the effective thermal conductivity is 0.56 W·m^{-1}·K^{-1} and the transient heat input is 100 seconds long (Figures 1-a and 1-b Note: measurement variability is in the order of ± 8%). The distance to boundaries must be increased when the specimen conductivity or the pulse duration increase. All measurements reported next were obtained using a ID=78 mm glass container and a transient heating period of 90 seconds. Measurements were repeated 9 times on each specimen; the probe was removed and the specimen was stirred and allowed to equilibrate with room temperature after each measurement.

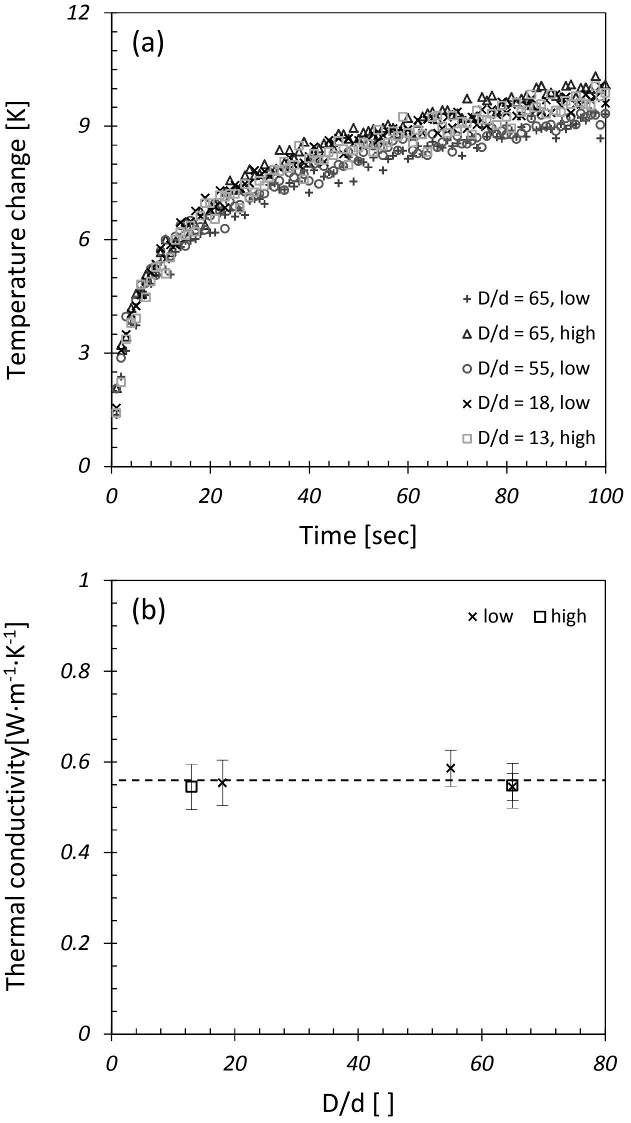

Fig. 1 Needle probe calibration and evaluation of boundary effects on gelated water.

3. RESULTS AND DISCUSSION

3.1 Air saturated binary mixtures

Binary mixtures of mono-size glass and lead particles 1mm in diameter were prepared at different solid volume fractions as indicated in Table 1. Results in Figure 2 show that the effective thermal conductivity of the granular mixture increases linearly with the volume fraction of the high thermal conductivity particles until $f_{\mathrm{lead}} < 0.8$; thereafter, there is a pronounced increase in thermal conductivity as $f_{\mathrm{lead}} > 0.8$. At low volume fractions of lead $f_{\mathrm{lead}} < 0.2$, the effective thermal conductivity is controlled by the low thermal conductivity glass particles; high conductivity particles are suspended in a matrix of glass particles and contribute very little to heat conduction. At intermediate lead volume fractions $0.2 < f_{\mathrm{lead}} < 0.6$, high conductivity grains form interconnected structures, but these are stochastic in nature and the data exhibits high variability (shaded area in Figure 2). At high lead volume fractions $f_{\mathrm{lead}} \geq 0.8$, percolating chains made of interconnected lead particles define the mixture heat conduction. Note that there is no marked percolation threshold given the low contrast in thermal conductivity among the pure granular phases (Mamunya et al. 2002). In general, results agree with previous studies (Gori et al. 2001; Okazaki et al. 1981). There are additional sources of variability, as hinted by the pure lead shot data, in part due to the lose packing at very low effective stress, and the particle size being similar to the probe diameter (Inaba 1986).

Since the P-wave velocity of lead is lower than that of glass ($V_{\mathrm{lead}} = 1900\ \mathrm{m \cdot s^{-1}}$ vs. $V_{\mathrm{glass}} = 3500\ \mathrm{m \cdot s^{-1}}$), phonon heat conduction is expected to be higher in glass. Yet, electron conduction prevails in lead and increases its conductivity above that of glass: solid-$\lambda_{\mathrm{lead}} = 35.3\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$ vs. solid-$\lambda_{\mathrm{glass}} = 1.05\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$. Despite the pronounced difference in thermal conductivity between solid-lead and solid-glass, the difference in the thermal conductivities of granular-glass and granular-lead is surprisingly small: granular-$\lambda_{\mathrm{lead}} = 0.32 \pm 0.05\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$ vs. granular-$\lambda_{\mathrm{glass}} = 0.16 \pm 0.005\ \mathrm{W \cdot m^{-1} \cdot K^{-1}}$. The higher thermal conductivity of lead shot indicates that while photon conduction is drastically reduced, it does still contribute to heat transfer.

Parallel conduction and series conduction models are plotted along with the experimental data in Figure 2, taking into consideration the data variability at lead fractions $f_{lead} = 0$ and $f_{lead} = 1$. Data trends plot closer to series conduction

Table 1 Binary mixture proportions; f_{lead} is the fraction of lead in the total volume of solids.

f_{lead} [ ]	*Glass*		*Lead*	
	M [kg]	V [m^3]×10^{-5}	M [kg]	V [m^3]×10^{-5}
0	*0.80*	*28.6*	*0.00*	*0*
0.2	*0.64*	*22.9*	*0.66*	*6*
0.4	*0.48*	*17.1*	*1.32*	*12*
0.6	*0.32*	*11.4*	*1.98*	*17*
0.8	*0.16*	*5.7*	*2.64*	*23*
1	*0.00*	*0.0*	*3.30*	*29*

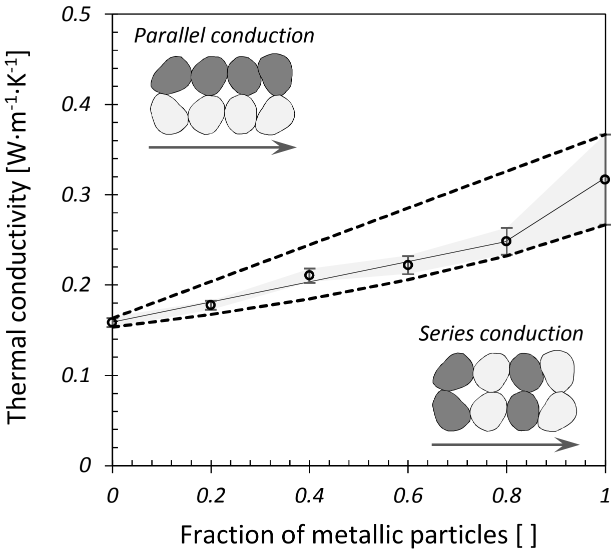

Fig. 2 Effective thermal conductivity of mono size air saturated binary mixtures. The theoretical upper and lower bounds defined by the parallel and series conduction models are presented for comparison.

$$\lambda_{parallel} = (1 - f_{lead})\lambda_{glass} + f_{lead}\lambda_{lead} \tag{5}$$

$$\lambda_{series} = \left[\frac{(1 - f_{lead})}{\lambda_{glass}} + \frac{f_{lead}}{\lambda_{lead}} \right]^{-1} \tag{6}$$

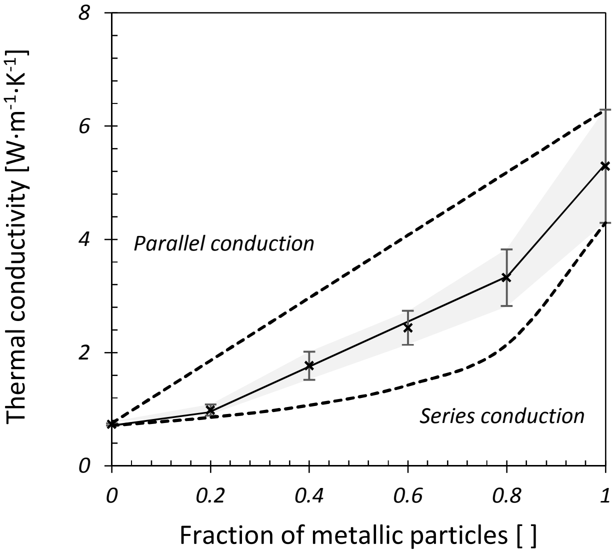

Fig. 3 Effective thermal conductivity of mono size water saturated binary mixtures. The theoretical upper and lower bounds defined by the parallel and series conduction models are presented for comparison.

3.2 Water saturated binary mixtures

Water-saturated mixtures were prepared by pouring grains into fluid-filled containers and removing entrained air using vacuum while stirring. Similarly to the air-dry case, water saturated binary mixtures show a quasi-linear increase in thermal conductivity with the volume fraction of lead shot (Figure 3), followed by a decisive increase when $f_{\mathrm{lead}} > 0.8$. Once again, measurement variability between $f_{\mathrm{lead}} = 0.2$ and 0.4 is a precursor to the onset of percolation. Parallel and series conduction bounds are superimposed on the figure.

3.3 Lead-shot in kaolinite paste

The thermal conductivity of kaolinite is 1.96 $\mathrm{W \cdot m^{-1} \cdot K^{-1}}$ (Zweifel et al. 2009); thus, when in suspension kaolinite particles act as high thermal conductivity colloids. A stable kaolinite-water paste was prepared at a water content twice the liquid limit w = 172%; the measured thermal conductivity is $\lambda_{\mathrm{paste}} = 0.94 \pm 0.07$ $\mathrm{W \cdot m^{-1} \cdot K^{-1}}$ which corresponds to twice the thermal conductivity of water and one half of the thermal conductivity of kaolinite at the liquid limit (Cortes et al. 2009).

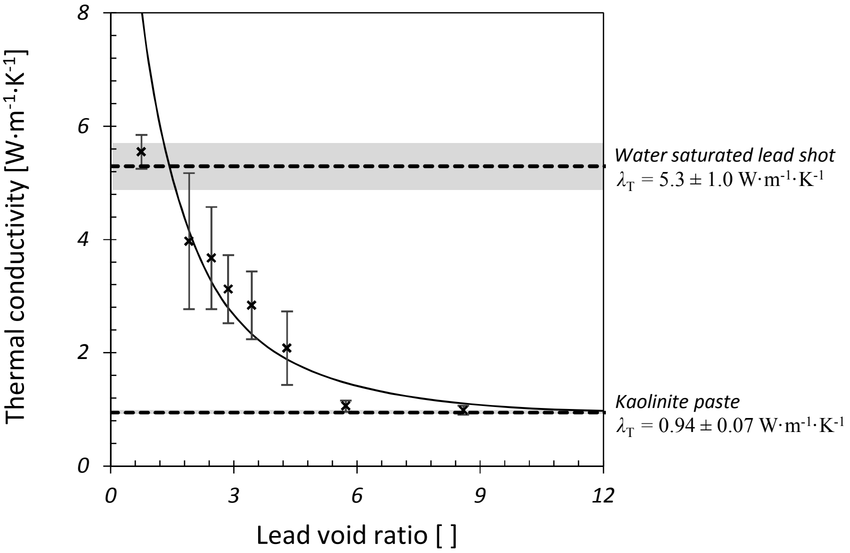

Figure 4. Effective thermal conductivity of kaolinite paste as a function of lead content. The thermal conductivities of water saturated granular lead and kaolinite paste are plotted for reference; the gray bands show the variability in the data.

Lead particles were added in equal increments and mixed in the kaolinite paste. The effective thermal conductivity data of the lead-kaolinite paste mixtures is best plotted against the lead void ratio e_{Pb} defined as:

$$e_{Pb} = \frac{V_k}{V_{Pb}} \qquad (7)$$

where V_{Pb} is the volume of solid lead and V_k is the volume of kaolinite paste (Figure 4). The lead void ratio decreases with an increase in the volume of lead particles. The asymptotic thermal conductivity for large e_{Pb} is λ_{paste}. The effective thermal conductivity and the measurement variability increase rapidly for values of e_{Pb} < 5 as interconnected lead particles start contributing to heat conduction. The thermal conductivity when lead shot is loosely packed (as in the case of air or water saturated - void ratio e_{Pb} = 0.75) is $\lambda_{\mathrm{Pb\text{-}paste}}$=5.55 ± 0.3 W·m^{-1}·K^{-1}.

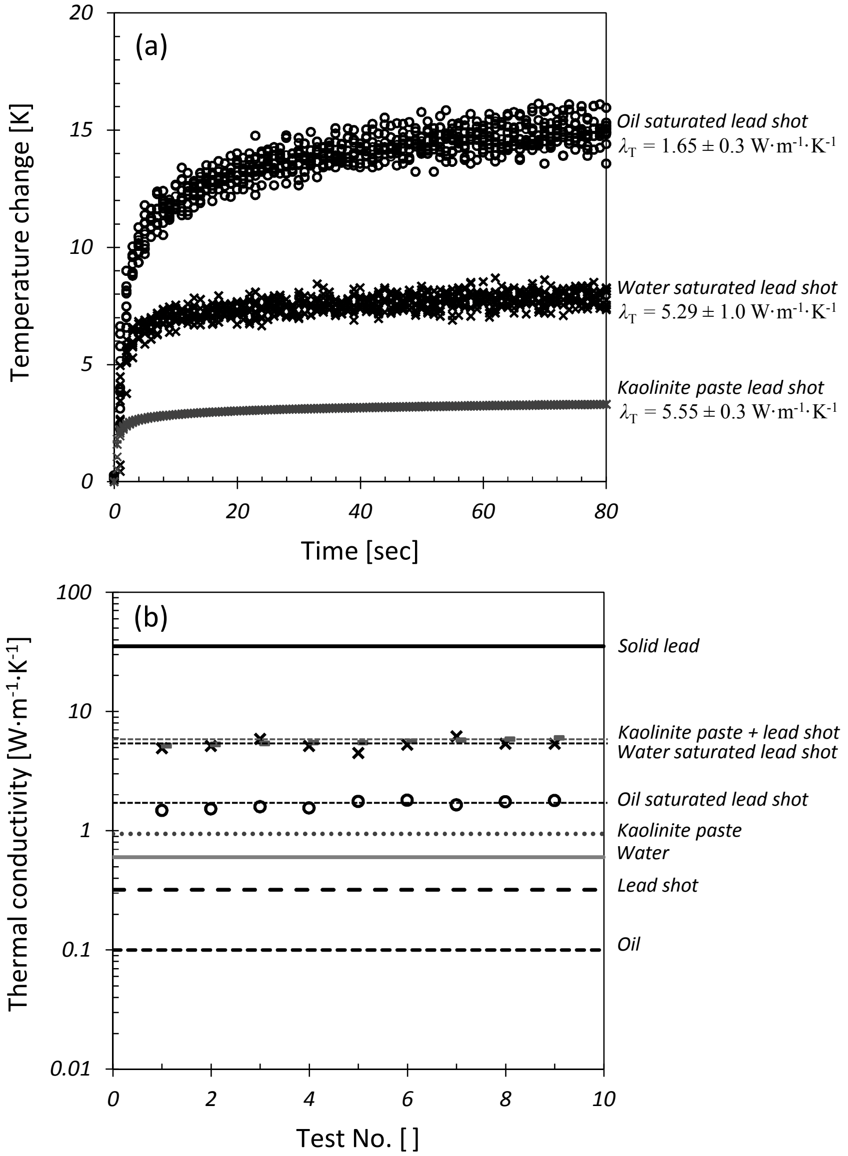

Fig. 5 Effective thermal conductivity of water and mineral oil saturated granular lead: (a) thermal signatures of all tests, and (b) effective thermal conductivity of the saturated granular media in comparison to the thermal conductivity of the individual phases.

3.4 Comparison: air, water, mineral oil and paste-saturated lead shot

At a lead void ratio e_{Pb}~0.72, the thermal conductivity of lead shot saturated with different fluids is $\lambda_{\mathrm{Pb\text{-}air}}$=0.32 ± 0.05 W·m^{-1}·K^{-1}, $\lambda_{\mathrm{Pb\text{-}oil}}$= 1.65 ± 0.3 W·m^{-1}·K^{-1}, $\lambda_{\mathrm{Pb\text{-}water}}$= 5.29

± 1.0 $W\cdot m^{-1}\cdot K^{-1}$, and $\lambda_{Pb\text{-}paste}$=5.55 ± 0.3 $W\cdot m^{-1}\cdot K^{-1}$. Figure 5 summarizes these results along with the thermal conductivities of the individual phases for comparison. Water saturated lead shot offers a more pronounced increase in thermal conductivity; however, mineral oil remains liquid up to temperatures as high as 200 ºC under atmospheric pressure and may be advantageous for high-temperature applications.

4. CONCLUSIONS

Thermal conductivity increases with the volume fraction of lead particles in binary mixtures of mono size glass and lead particles. A decisive increase in thermal conductivity for high volume fractions $f_{\text{lead}} > 0.8$ suggests the formation of a post-percolation conductive structure. Conversely, high data variability at intermediate volume fractions $0.2 < f_{\text{lead}} < 0.6$ indicate the formation of interconnected chains of conductive particles that are precursors to percolation. The thermal conductivity of mixtures tracks closer to the series conduction bound.

The thermal conductivity of lead shot is higher than the thermal conductivity of glass beads, despite the higher p-wave velocity of glass particles. Hence, photon heat conduction prevails at high lead volume fraction.

The effective thermal conductivity of air saturated loosely packed lead shot increases 5 times or 16.5 times when either oil or water is the saturating pore fluid respectively.

Suspended kaolinite particles increase the thermal conductivity of water by a factor of 2 at a water content equal to twice the liquid limit. Suspended lead grains start affecting the conductivity in kaolinite paste when the lead void ratio is less than e_{Pb}<5.

The average effective thermal conductivity of lead filled with kaolinite paste is 5.55 ± 0.3 $W\cdot m^{-1}\cdot K^{-1}$, indicating that the beneficial effect of the higher conductivity pore fluid can be lessened by an increase in particle-to-particle heat conduction resistance induced by the paste coating.

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