The 2016 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM16)
Jeju Island, Korea, August 28-September 1, 2016

A new thinking of the critical state of slope failure

*Yan-Peng Zhu[1)], Xiao-Yu Yang[2)]
Xiao-Rui Ma[3)], Jing-Bang Li[4)]
Xiao-Hui Yang[5)]

[1), 2,), 3), 4), 5)] College of Civil Engineering, Lanzhou University of Technology, Lanzhou 730-050, China
[1)] zhuyp1@163.com

ABSTRACT

In the slope analysis progress, the definition of factor of safety was usually given by the concept of strength reservation, which is more rational than others. From this view, solving initial strength and critical strength can get the value of Fos. In this paper, using the concept of strength area to represent initial and critical strength, and a slope example calculated by FEM shows that the critical state is independent on initial parameters, is the inherent attribute of slope. Meanwhile, utilizing the double strength reduction method can get several paths of reduction, finally will produce some various critical points, and fitting these points can get a straight line, which conforms to Mohr-Coulomb criterion. From this view, the critical state is not related to the initial strength parameters, but the value of safety factor is determined by initial strength and critical strength, the geometric model of slope controls the ability to resist unstable failure. Using these concepts can better understand and analyze the stability of slope.
Key words: finite element method; strength reduction method; critical state of slope

1. INTRODUCTION

The stability of high fill slope is an important research problem, finite element strength reduction method is widely applied to evaluate this problem. The method for the definition of safety factor is usually based on the concept of safety reserves. (Duncan

---

[1)] Professor

[2)] Graduate Student

1996) proposed that the factor of safety could be understood as the ratio of the initial strength and critical strength. And many scholars also believe safety factor based on the concept of strength reserves is the most reasonable way. (Zhao 2007) pointed out the other ways of the definition of defects.

(Tang 2007) considered that the traditional strength reduction method's (T-SRM) strategy of the internal friction angle and cohesion adopted the same reduction coefficient, which does not conform to the objective fact, and proposed a new method based on dual parameters of reduction, meanwhile, pointed out the existence basis; (Yuan 2013) and (Bai 2013) also researched on double reduction method, which was mainly about the determine of the expression of safety factor. In fact, the reduction path of DRM is different from T-SRM's path and the critical strength points obtained also are different. But these two methods are all based on the linear reduction theory, initial strength parameters of slope in the calculation are known, so the critical state of slope failure determines this difference, according to the locations of critical points can determine the distribution law of critical state, which should comply with the Mohr-Coulomb failure criterion.

2. SAFETY FACTOR OF DRM

The safety factor of slope is unique, but the double reduction method has two reduction factors, which bring the difficulty for determination of safety factor. The mean, the minimum is lack of theoretical basis, at present.

For the convenience of calculation, the DRM calculation process is briefly described here. Firstly, individually reducing the internal friction angle can determine the reduction factor SRF_{φ}', then reducing the cohesion can get the reduction factor SRF_{c}', and using these two reduction factors can determine the value of reduction ratio K as shown in Eq. (1)

$$K = \frac{SRF_{\varphi}'}{SRF_{c}'} \tag{1}$$

Next, considering the coexistence of the internal friction angle and cohesion, in the reduction process, always ensuring the ratio of the internal friction angle and cohesion conform to the K already calculated by Eq. (1). Until the slope is unstable, two final reduction factors can be determined, one is SRF_{φ}, and the other one is SRF_{c}.

The coordinate of strength parameters is illustrated in Fig. 1. There are two points in this coordinate, one is initial strength parameter point, such as A, and the other one is critical failure state point B as shown in Fig. 1

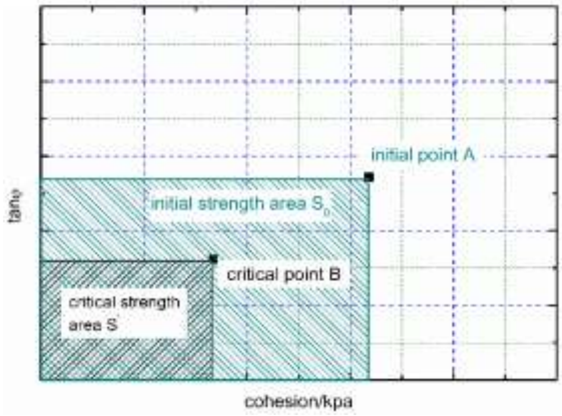

Fig. 1 Two kinds of points and areas

For T-SRM, its reduction factors and safety factor should comply with Eq. (2), Eq. (3) and Eq. (4)

$$FOS = SRF_{\varphi} = SRF_{c} \tag{2}$$

$$FOS^2 = SRF_{\varphi}^{\ 2} = SRF_{c}^{\ 2} \tag{3}$$

$$FOS^2 = SRF_{\varphi} \cdot SRF_{c} \tag{4}$$

For the definition of reduction of T-SRM, its two reduction factors are equal which is shown in Eq. (5)

$$SRF_{\varphi} = \frac{\tan\varphi_0}{\tan\varphi'} = SRF_{c} = \frac{c_0}{c'} \tag{5}$$

Next, we can get the expression of Eq. (6)

$$FOS^2 = \frac{\tan\varphi_0}{\tan\varphi'} \cdot \frac{c_0}{c'} \tag{6}$$

In the Cartesian coordinates, the product of the horizontal and vertical coordinates is the value of rectangle area. The initial point will lead to an initial strength area S_0, Eq. (7). Likewise, the critical point will get a critical strength area S', Eq. (8)

$$S_0 = \tan\varphi_0 \cdot c_0 \tag{7}$$

$$S' = \tan\varphi' \cdot c' \tag{8}$$

Eq. (9) is determined by Eq. (6), Eq. (7) and Eq. (8)

$$FOS^2 = \frac{\tan\varphi_0 \cdot c_0}{\tan\varphi' \cdot c'} = \frac{S_0}{S'} = SRF_{\varphi} \cdot SRF_{c} \tag{9}$$

Above all, these relation formulas are right for T-SRM. However, (BAI 2013) proposed that traditional strength reduction method just was a special case of DRM. That is, when K is 1 in DRM, the definition of the reduction factor is same as the SRM's.

From special situation to general situation, in this here, the author puts forward the concept of “strength reserve area”. In the coordinate system of strength parameters, Fig. 1, initial strength parameter point A and the origin of the coordinate (0,0) consist of the rectangular section. The product of the vertical and horizontal coordinates is rectangular area which is Cartesian coordinates’ nature, and is no conditions for the establishment. Then, in the same way, the rectangular area S' is made by the critical state point B and the point of origin (0,0) as shown in Fig. 1.Moreover, it should be pointed out that, for the area, it is not the intensity, it only represents the product, in the coordinate system, which has only the geometric meaning. The numerator and denominator of expression in the strength parameters coordinate system respectively represent the “initial strength area” and “critical strength area”, by using this view, Eq. (9) also is applicable for DRM. So the FOS of DRM is solved by using Eq. (10) as follows.

$$FOS = \sqrt{SRF_{\varphi} \cdot SRF_{c}} = \sqrt{\frac{S_0}{S'}} \tag{10}$$

3. CRITICAL STATE OF FAILURE

3.1 Intrinsic properties of slope

The nature of slope unstable failure is the attenuation of shear strength of soil, and the safety factor determined by the notion strength reserve can reflect this concept. The definition of safety factor was proposed in (Duncan 1996) Eq. (11). τ_0 indicates initial strength of slope, τ' indicates critical strength.

$$FOS = \frac{\tau_0}{\tau'} \tag{11}$$

In fact, if adopting this view, the definition of safety factor for the idea of strength reserve, can be divided into two parts, one part is to solve the initial strength, and the other part is to determine the critical state. And the initial parameters are known, so the research on the distribution law of critical points is very important.

Mohr-Coulomb failure criterion is widely used calculate the slope stability, which depicts the condition of material failure. In author’s opinion, if the failure criterion is determined, the critical state of slope also is confirmed. In general situation, Mohr-Coulomb can be regarded as straight line, Eq. (12)

$$\tau' = c' + \sigma \cdot \tan\varphi' \tag{12}$$

If this equation is in the coordinate of $\tan\varphi - c$,we can get the expression as shown in Eq. (13)

$$\tan\phi' = -\frac{1}{\sigma} \cdot c' + \frac{\tau}{\sigma} \tag{13}$$

Eq. (12) reveals that the critical strength τ' is not related to c_0 and $\tan\varphi_0$, this view also conforms to the aspect of the failure criterion in plastic mechanics. So, the author thinks that the distribution law of critical points is the inherent attribute of slope, initial strength parameters have no influence on the critical state, and this critical state merely is determined by φ' and c'. Initial strength parameters only effect the value of safety factor.

3.2 Quantitative calculation

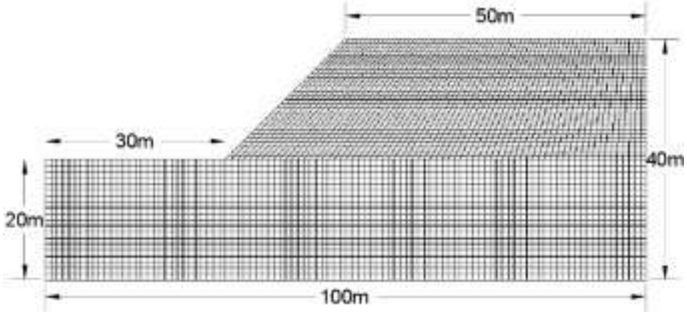

Fig. 2 geometrical model of example

Geometrical model of example is illustrated in Fig. 2, the value of safety factor approximate is 1.2. The strength parameters are given in the Table 1. Using finite element software of ADINA (version 9.0.6) to calculate the safety factor. And in this here, the example is given three different groups of initial strength parameters to prove the viewpoint of this paper. The Mohr-Coulomb yield criterion is adopted, unassociated flow criterion and the critical state criterion is calculation non-convergent, and the displacement convergence criterion is taken into account, and the convergence tolerance is 0.001.

Table 1 Strength parameters of example 2

Group	*E* /kPa	ν	*c* /kPa	φ /°	γ/kN/m^3
1	10^5	0.3	42	17	20
2	10^5	0.3	45	17	20
3	10^5	0.3	42	22	20

The calculation method is DRM, which will provide four sort safety factors, singly reducing the internal friction angle to finish the calculation, individually reducing the cohesion, and T-SRM, the last way is DRM. The calculation results are illustrated in Table 2, Table 3, and Table 4. The results of reduction factor and safety factor are determined by Eq. (10). And the images of horizontal plastic strain are illustrated in Fig. 3, Fig. 4, and Fig. 5.

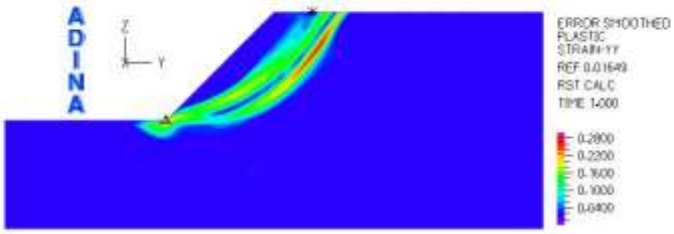

Fig. 3 Horizontal plastic strain of group1

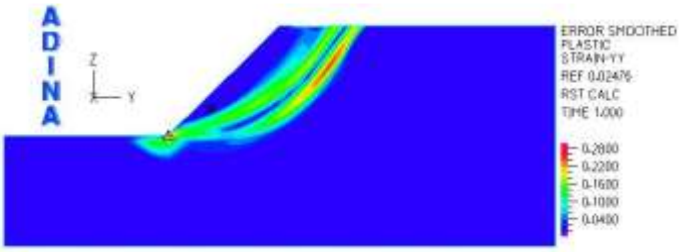

Fig. 4 Horizontal plastic strain of group 2

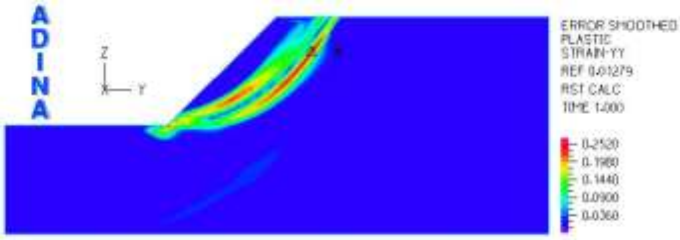

Fig. 5 Horizontal plastic strain of group 3

Table 2 The results of Group 1

Group 1	tan φ'	c'	SRF_{φ}	SRF_{c}	*FOS*
SRM	33.6000	0.2446	1.25	1.25	1.25
Reducing φ	42.0000	0.1584	1.93	1.00	1.39
Reducing c	27.8146	0.3060	1.00	1.51	1.23
DRM	36.8421	0.2098	1.46	1.51	1.29

Table 3 The results of Group 2

Group 2	tan φ'	c'	SRF_{φ}	SRF_{c}	*FOS*
SRM	34.6154	0.2350	1.30	1.30	1.30
Reducing φ	45.0000	0.1347	2.27	1.00	1.51
Reducing c	27.7778	0.3060	1.00	1.62	1.27
DRM	38.7931	0.1881	1.16	1.63	1.37

Table 4 The results of Group 3

Group 3	tan φ'	c'	SRF_{φ}	SRF_{c}	*FOS*
SRM	29.5775	0.2850	1.42	1.42	1.42
Reducing φ	42.0000	0.1584	2.56	1.00	1.60
Reducing c	20.0000	0.4040	1.00	2.10	1.45
DRM	32.3077	0.2559	1.30	1.58	1.43

The distribution of these critical points in the coordinate approximately conforms to the straight line, and the fitting analysis can get the expression of this line. The fitting calculation information of fitting is shown in Table 5 and the fitting line of critical points is illustrated in Fig. 6. The Adj. R-Square is 0.99463 which reveals the result of fitting is acceptable.

Table. 5 The information of fitting

intercept		slope		statistics
value	Standard error	value	Standard error	Adj. R-Square
0.6056	0.00825	-0.01068	2.36409E-4	0.99463

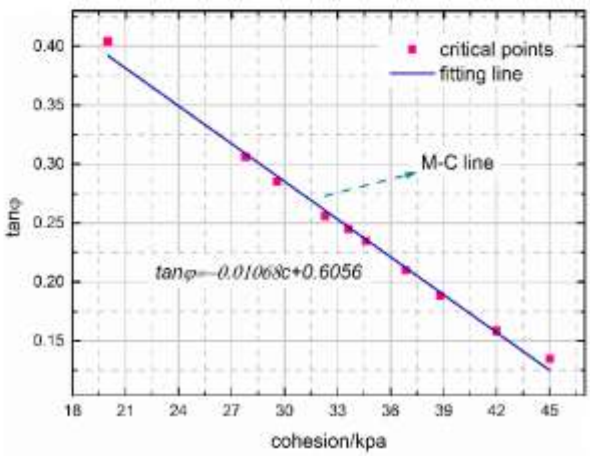

Fig. 6 Fitting curve

The distribution law of critical points conforms to Eq. (13) in the coordinate of $\tan\varphi - c$, the detail expression is shown in Eq. (14)

$$\tan\varphi = -0.01068 \cdot c + 0.6056 \tag{14}$$

The result reveals if the line equation is determined, all the slope critical points are located in this M-C line in the $\tan\varphi - c$ coordinate. And the initial strength parameters do not affect the distribution of critical failure points. Combining the concept of strength area with Eq. (10) points out if the critical strength area S' is determined, the slope stability also is determined since the initial parameters for the same slope model is known. Thus all the critical points comply with the equation of critical line, the objective function is how to solve the value of Eq. (8). The function of example slope is shown in Eq. (15) which is determined by Eq. (14) and Eq. (8)

$$S' = -0.01068 \cdot c^2 + 0.6056 \cdot c \tag{15}$$

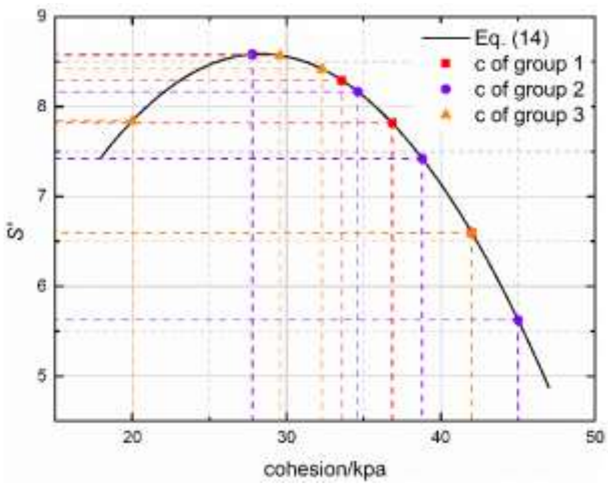

Fig .7 The curve of objective function Eq. (14)

The Fig. 7 shows that the change law of S' with *c*, which is a quadratic functions. And the maximum value $S'_{\max}$ is 8.585 with *c* is 28.352 kpa. No matter what the initial value is, the critical failure points are all located in the critical line. This new thinking is convenient for evaluating the slope stability and determining the value of safety factor. Of course, the value of safety factor is related to the initial strength parameters.

4. CONCLUSIONS

(1) The reduction factor is different from the slope safety factor, for the DRM, the problem of determination of safety factor is an important issue and can't afford to ignore, this paper proposes the equation of safety factor with reduction factor, which have theoretical significance, is rational.

(2) The critical strength points is not related to the initial strength parameters, for a known slope, the critical state is the inherent property of the slope. Using the concept of strength area to calculate the safety factor, and the critical fitting line complies with the Mohr-Coulomb strength line in the $\tan\varphi - c$ coordinate system, which can get the objective function of S'. Using this function can quantitatively determine the slope critical state, which is convenient for analyzing the slope stability and the value of safety factor.

REFERENCES

Duncan J M. (1996). "State of the art: limit equilibrium and finite-element analysis of slopes". *J. Journal of Geotechnical and Geoenvironmental Engineering，ASCE*, **122**(7): 577－596.

Zhao Shang Yi，Zheng Ying Ren, Shi Wei Min, et al.(2002). "Analysis on safety factor of

slope by strength reduction FEM”. *J. Chinese Journal of Rock Mechanics and Engineering*, **24**(3): 343－346.
Tang Fen, Zheng Ying Ren, Zhao Shang Yi. (2007). “Discussion on two safety factors for progressive failure of soil slope”. *J*. *Chinese Journal of Rock Mechanics and Engineering*, **26**(7): 1402－1407
Yuan Wei, Bai Bing, Li Xiao Chun, et al. (2013). “A strength reduction method based on double reduction parameters and its application”. *J. Journal of Central South University*, **20**(9): 2555－2562.
Bai Bing, Yuan Wei, Li Xiao Chun. (2014). “A new double reduction method for slope stability analysis”. *J. Journal of Central South University*, **21**(3): 1158－1164.