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Shakedown analysis of pile foundation with limited plastic deformation

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ABSTRACT

A general method to restrain the plastic deformations of a vertical long pile in cohesionless soil when subjected to cyclic lateral loading is introduced. For this purpose shakedown theorem is developed and applied to the structure. One of the main requirement of the elasic-plastic analyses and design processes is take into consideration the plastic residual deformations of the structures, however the shakedown theorem gives no information concerning the amount of the plastic displacements, and consequently residual strains cumulate in the structures. In this study the complementary energy for the internal residual forces is purposed as an overall criterion of the plastic deformations of the pile structures and the residual strains are constrained by applying a boundary for the amount of this energy. Furthermore, because of the uncertainties the boundary condition on the complementary energy is given randomly, reliability index cocept is used to introduce the so-called strict safety index and critical stresses updated during the iteration. Limit load strength of lateral loads on the given probabilistic conditions are calculated. Plastic limit state domains are illustrated for shakedown load parameters. The reliability-based-problem evaluated by the use of an algorithm for nonlinear programming. Moreover, for comparition and verification, pile foundations were numerically modeled by using finite element code ABAQUS.

Keywords: shakedown theorem, lateral piles, reliability-based-problems, load parameter.

1. INTRODUCTION

Pile-soil interaction has been the subject of several studies in the last decades. Poulos (1982) proposed that depending upon the type of cyclic loading, there are two main phenomena that may lead to the increased deformation of laterally loaded piles:

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(i) Structural shakedown of the laterally loaded piles when the deformations stabilize; otherwise, deformations accumulated which leads to collapse.
(ii) The stiffness and strength of the soil might be decreased because of the cyclic loading.

Qin and Guo (2014) introduced the nonlinear response of lateral loaded rigid piles in sandy soil by applying elastic-plastic solutions. They provided critical parameters for the limiting force profile which are useful in nonlinear analysis of laterally loaded piles. Keawsawasvong and Ukritchon (2016) used a numerical solution to determine the ultimate lateral load capacity of rectangular pile in clay. Giannakos et al. (2012) presented a nonlinear constitutive model for the cyclic behavior of piles in dry dense sand and numerical results were compared with experimental results. Mucciacciaro and Sica (2018) studied the mechanical behavior of undrained clay by introducing a kinematic hardening model.

Over the years two approaches have been improved to evaluate the uncertainties in engineering structures. The first method is the deterministic design, in which a global factor of safety or a load factor is applied. The second method is the reliability-based design, when the information in respect to the design is known to be within certain limits and have recognized distributions of probability. Although the deterministic design has been favorably used for decades, the appropriate safety is unclear for a given factor of safety. In the reliability-based design, the uncertainties are defined by randomly distributed variables, in which the contribution of occurrence of each feasible value of the variable is examined and the most frequent values of a random variable are related with the highest amounts in the probability density function. In geotechnical engineering the uncertainties have a very extensive role and demand intensive computations. Wang and Cao (2013) developed a reliability based design method for drilled shafts with applying Monte Carlo Simulation. Klammler et al. (2013) proposed an approach for introducing a pile driving criteria for individual pile foundations under axial loading. Li et al. (2015) based on the reliability design method calibrated the resistance parameters for the design of piles foundations.

By the use of elastic-plastic analysis and design approaches, considerable saving in material can be achieved. As a result of this advantage, however excessive residual displacements and large plastic deformations could develop, which might lead to failure of the structure. Over the years various limit theorems for the residual displacements and plastic deformations have been recommended in the literature (see e.g. Tin-Loi (2000), Liepa et al. (2016); Weichert and Maier (2002) and Levy et al. (2009)). In this paper a proper computational method presented when the complementary energy defined as a general bound for the plastic performance of the structures and the plastic deformations need to be constrained by considering an appropriate limit for the amount of the complementary energy (Kaliszky and Lógó 1997 and Movahedi and Lógó 2011).

2. Elastic-plastic modeling of the pile foundation

Broms (1964) proposed that short and long piles have different failure modes. A short free head pile rotates or tilts to a point located close to its toe and passive resistance extends above and below the point of rotation. For long free head pile, the passive resistance is large and pile cannot rotate or tilt. The lower part stays almost vertical and the upper portion deflects in flexure. Failure occurs when the maximum bending moment exceeds the yield strength of the pile section and a plastic hinge forms at the point of maximum bending moments as shown in Fig. 1.

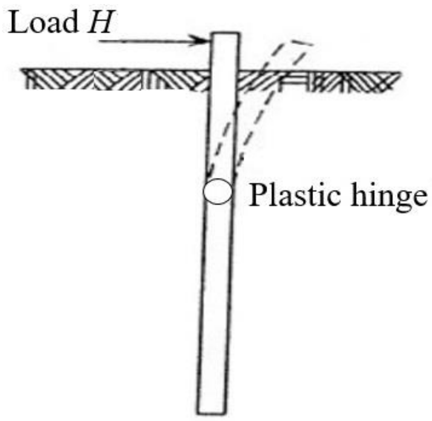

Fig. 1 Failure mode of the long free head pile under horizontal load

Consider a constant pile cross section for a free head long pile, a plastic hinge with a plastic moment of M^p will form at a depth l which has no shear force. The M^p can be calculated using elastic-plastic solutions proposed by Guo (2006)

$$\frac{M^p}{A_L} = \frac{1}{n+2}\left[\alpha_0^{n+1} + (n+1)\frac{H}{A_L}\right]^{\frac{n+2}{n+1}} - \left(\frac{\alpha_0^{n+2}}{n+2} + \frac{\alpha_0 H}{A_L}\right) + \frac{M_e}{A_L} \quad (1)$$

Here $A_L = \gamma' N_g D^{1-n}$associated with the limiting force profile for cohesionless soil, γ' effective density of overburden soil, N_g gradient to correlate soil undrained strength, D the outer diameter of an cylinder pile, n power for the limiting force profile, α_0 equivalent depth to consider the resistance at the ground surface; H lateral load and $M_e = He$, e the distance from lateral load to the mudline.

The instructions to determine the values of the parameters are proposed by Guo (2006, 2012) and limiting force profile are considered as advised for cohesionless soil by Broms (1964).

3. Residual plastic limit theorem

Let us suppose that the structure has been defined by the concept of plastic analysis and design methods. Accordingly by applying load P_0 internal plastic forces Q^p will appear in the structure. When the load is decreased under unloading elastic deformations occur and then the elastic internal forces $-Q^e$ will take place in the structure. Therefore after complete unloading the residual forces will remain in the structure.

$$Q^r = Q^p - Q^e \quad (2)$$

Where

$$Q^e = F^{-1} G^T K^{-1} P_0 \quad (3)$$

Here F is the flexibility matrix, G denotes the geometrical matrix, K is the stiffness matrix. Let us suppose the positive-definite function, the complementary energy can be determined from the residual forces.

$$C_r = \frac{1}{2}\sum_{i=1}^{n}\frac{1}{S_i}\int_0^{l_i}\left(Q_i^p(s) - Q_i^e(s)\right)^2 ds \geq 0 \tag{4}$$

Here $Q_i^e(s)$ and $Q_i^p(s)$ are the functions of elastic and plastic internal forces, $l_i, (i = 1, 2, \dots, n)$ denotes the length of the members, S_i expresses tensile stiffnesses and flexural stiffnesses for trusses and beam members respectively.

A proper computational method proposed that the complementary energy for the internal residual forces could be defined as a general measure of the plastic performance of the structures and the residual deformations need to be constrained by considering a limit for the amount of this energy (Kaliszky and Lógó 1997 and Movahedi and Lógó 2011).

$$\frac{1}{2}\sum_{i=1}^{n}\frac{1}{S_i}\int_0^{l_i}\left(Q_i^p(s) - Q_i^e(s)\right)^2 ds - C_{r0} \leq 0 \tag{5}$$

Where C_{r0} is a proper permissible energy value for C_r. Now let us consider the case of beam elements

$$C_r = \frac{1}{2E}\sum_{i=1}^{n}\frac{1}{I_i}\int_0^{l_i}(M_i^r(s))^2 ds \tag{6}$$

the complementary energy computed from the residual forces. Here $l_i, (i = 1, 2, \dots, n)$ denotes the length of the beam members, I_i is the moment of inertia of the beam elements, $M_i^r(s)$: the residual moment of the beam members, E: the Young's modulus. In case of the moments M_{i1}^r and M_{i2}^r applying at the ends of the members the integral function in Eq. (6) can be expressed as:

$$\int_0^{l_i}(M_i^r(s))^2 = \frac{1}{3}[(M_1^r)^2 + (M_1^r)(M_2^r) + (M_2^r)^2 \tag{7}$$

Applying Eq. (7) the plastic deformations are limited if an appropriate limit value C_{r0} is introduced.

$$\frac{1}{6E}\sum_{i=1}^{n}\frac{l_i}{I_i}[(M_1^r)^2 + (M_1^r)(M_2^r) + (M_2^r)^2] - C_{r0} \leq 0 \tag{8}$$

4. Reliability-based residual plastic limit theorem

Let us assume that X_R introduces the non-negative bound for the statically admissible forces X_S with probability density functions $f_R(X_R)$ and $f_S(X_S)$, respectively.
The probability of failure can be determined by the following equation:

$$P_f = P[X_R \leq X_S] = \iint_{X_R \leq X_S} f_R(X_R)\, f_S(X_S) dX_R dX_S. \tag{9}$$

An alternative formulation of the above problem is in term of the so-called limit state function expressed by

$$g(X_R, X_S) = X_R - X_S \leq 0. \tag{10}$$

Let us suppose that the constrains on the complementary energy for the internal residual forces is given by uncertainties and it follows the normal distribution function with the given mean value $\bar{C}_{r0}$ and S_w variance. The probability of the failure function can be determined in the following form:

$$P_{f,calc} = \int f(\bar{C}_{r0}\,, S_w) dx. \tag{11}$$

Using the strict safety index a reliability boundary condition can be formed:

$$\beta_{target} - \beta_{calc} \leq 0 \tag{12}$$

where β_{target} and β_{calc} are calculated as follows:

$$\beta_{target} = -\Phi^{-1}\left(P_{f,target}\right); \tag{13}$$
$$\beta_{calc} = -\Phi^{-1}\left(P_{f,calc}\right). \tag{14}$$

In Eqs. (13 and 14) Φ is cumulative distribution function of the normal distribution function.

5. Shakedown analysis and design methods

Using load parameters $m_1 \geq 0$, $m_2 \geq 0$ in Fig. 2 when a long pile in cohesionless soil subjected to two separate constant loads P_1 and P_2. For each loading combination $W_i = [m_1 P_1,\ m_2 P_2]$ a shakedown load parameter m_{sh} can be calculated. Using these parameters a limit state curve can be created in $m_1$and m_2 plane as shown in the numerical example (Fig. 4).

Considering admissible bending moment fields M_j a statically admissible stable shakedown load parameter m_{sh} can be achieved from the condition that even the maximum bending moment does not overstep the fully plastic moment, i.e. $max\left|M_j\right| \leq M^P$.

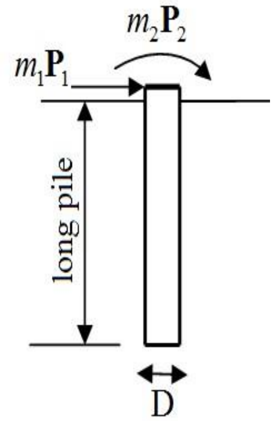

Fig. 2 The combination of loads on the pile

The solution approach based on the static theorem of shakedown analysis, therefore M^r satisfies the equilibrium equation

$$GM^r = 0 \tag{15}$$

Eq. (15) certifies that during the loading the structure will not undertake unlimited plastic displacements however, doesn't give us any information concerning permanent displacements which remain in the structure after shakedown. In order to restrict the permanent displacements, complementary energy for the internal residual forces is defined as a general measure of the elastic-plastic performance of the structures and the residual deformations are constrained by Eq. (8).

The elastic internal moment force equation can be formed from Eq. (16):

$$M^e = F^{-1}GK^{-1}m_{sh}W_i \tag{16}$$

Eq. (17) defines the yields condition, the M^p can be calculated using Eq. (1) which proposed by Guo (2006).

$$-M^p \leq M^r + maxM^e \leq M^p \tag{17}$$

For deterministic method residual plastic deformations of the pile structures are bounded with applying permissible energy value C_{r0}:

This mathematical optimization problem can be executed by the use of nonlinear algorithm.

$$\left.\begin{array}{c} m_{sh} = max \\ GM^r = 0 \\ -M^p \leq M^r + maxM^e \leq M^p \\ M^e = F^{-1}GK^{-1}m_{sh}W_i \\ \frac{1}{6E}\sum_{i=1}^{n}\frac{l_i}{I_i}[(M_1^r)^2 + (M_1^r)(M_2^r) + (M_2^r)^2] - C_{p0} \leq 0 \end{array}\right\} \tag{18}$$

Considering probabilistic method the bound can be defined with introducing safety index β:

$$\left.\begin{array}{c} m_{sh} = max \\ GM^r = 0 \\ -M^p \leq M^r + maxM^e \leq M^p \\ M^e = F^{-1}GK^{-1}m_{sh}W_i \\ \beta_{target} - \beta_{calc} \leq 0 \end{array}\right\} \quad (19)$$

6. Numerical example

To evaluate the shakedown theories and solution methods explained above, a mathematical nonlinear programming procedure is developed. The application of the theories is figured out by a numerical example. The example shows a free head long pile when lateral load and bending moment are acting at its top with diameter of $D = 0.4m$ (Fig. 3). The constant loads are $P_1 = \mathrm{H} = 10KN$, $P_2 = \mathrm{M} = 24KNm$, the flexural stiffness $EI = 117.47MNm^2$. The vertical pile embedded in cohesionless soil, the density of soil is$1900\ kg/m^3$; Young's Modulus $= 170\ MPa$ poisson's ratio $= 0.3$ and friction angel $= 41^0$.

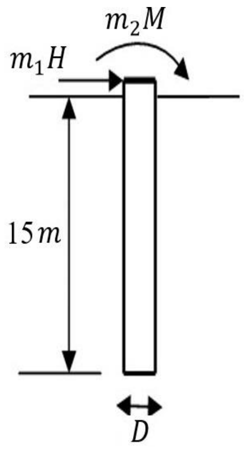

Fig. 3 Loads on the free head pile

The results are presented in Fig. 4. One can see the limit load state domains in two different functions for deterministic and probabilistic problems. For deterministic problem the limit load state domains are considered in the case of different complementary energy value forC_{r0}. In the case of probabilistic problem the limit load state domains are illustrated for different mean values $\bar{C}_{r0}$ when variance $S_w = 3$, target safety index $\beta_{target} = 3.2$ and the given probability of failure $P_f = 0.00069$.

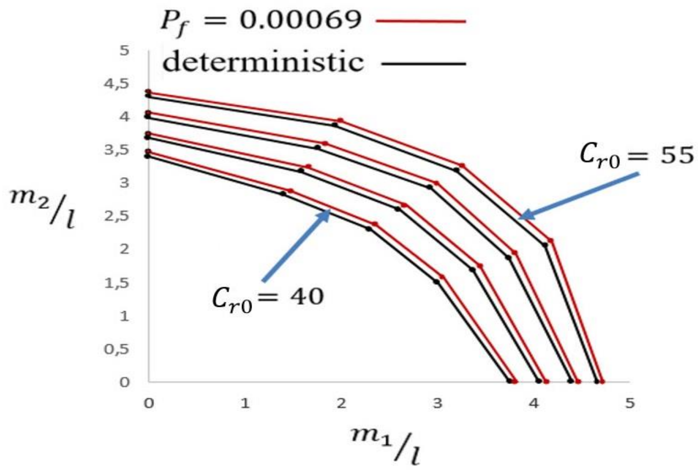

Fig. 4 Limit load state domain for free head long pile

7. Finite element model

The accuracy of the given example numerically verified by using finite element code ABAQUS in 3D. The pile is considered with 3D beam elements and cohesionless soil replaced by 8-node brick elements, the entire 3D model domain was $50m \times 50m$ and vertical height $25m$ as illustrated in Fig. 5. In each model approximately $34{,}000$ elements were used. The pile structure were subjected to one way cyclic loading following the suggestions of Long et al. (1994) that, the one-way cyclic lateral loads causes more permanent deformations and bigger cumulative strains than the two way lateral loads. The soil behavior is described by Drucker-Prager model and the behavior of pile assumed to be elastic-plastic. The geostatic stress created during the first step of the finite element analysis. Pile and soil connected by “tie” contact according to a master and slave definition.

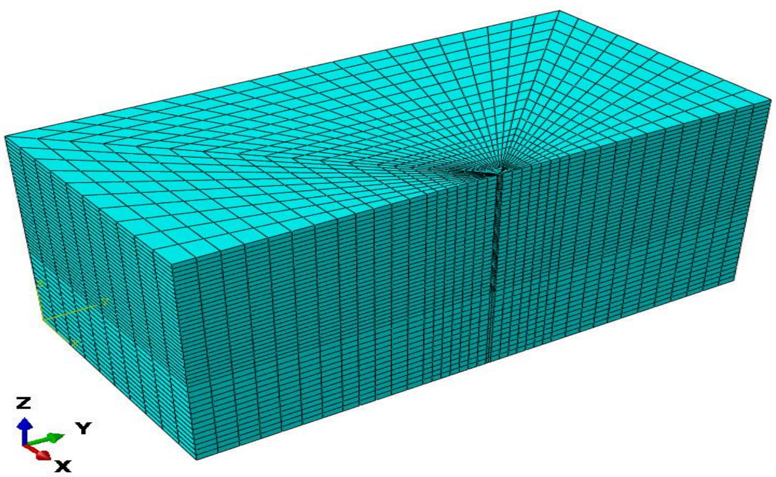

Fig. 5 3D view of Finite element modeling.

Conclusions

In this paper elastic-plastic shakedown method is described to determine the limited lateral load capacity of the long piles in cohesionless soil. The complementary energy is applied as an overall criterion of the plastic behavior of the pile structures. The limit on the complementary strain energy is given randomly, safety index concept is used to introduce the so-called strict safety index. Plastic limit curves are illustrated for the shakedown load parameters. Furthermore, for comparition and verification, pile foundations were numerically executed by using finite element code ABAQUS. The results of numerical example shows that the given mean values and probability of failure have influence on the plastic deformation of the pile structure.

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