The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Study on structural dynamic behaviors of Hagia Sophia through micro-tremor measurement

*Kota Yamaoka[1)], Takashi Hara[2)] and Kenichiro Hidaka[3)]

[1)] *World Heritage Studies, University of Tsukuba, Ibaraki 305-0821, Japan*
[2)] *Department of Civil Engineering and Architecture, Tokuyama College of Technology, Yamaguchi 745-8585, Japan*
[3)] *Faculty of Fine Arts, Tokyo University of the Arts, Tokyo 110-8714, Japan*
[1)] m12_yamaoka@heritage.tsukuba.ac.jp

ABSTRACT

Hagia Sophia, the well-known domed complex built by Justinian the Great in the 6th century in his Capital, Constantinople, the present Istanbul, suffers from structural deformation and material decay, which have been caused by repeated seismic attacks and which accumulated through centuries down to the present. It is imperative to understand its structural behavior not only in the static phase but also in the dynamic phase.

Our results are based upon the data of the natural frequencies and the mode shapes, which were obtained through micro-tremor measurement in situ, using the frequency domain decomposition method. Of particular importance are the inconsistent vibrations shown in the uppermost part of the north main arch. The structural analysis of Hagia Sophia, should take due consideration of the asymmetry of the structure. The results of our measurements may contribute to the further clarification of the difference between the northern and the southern arch, which were treated to be symmetrical for the sake of structural simplicity in the past analyses.

1. INTRODUCTION

Since the static analysis of Mainstone (1988), many studies on the structural characteristics of Hagia Sophia have been developed such as the elastic-stress analysis and input seismic motion implemented by Cakmak (2009), who obtained the maximum principal stress distribution of Hagia Sophia. The structural weak-points were identified by Ozkul (2007) by inputting seismic motion to a linear dynamic analysis.

These numerical analyses, however, were based upon mathematical models, which are purely symmetrical and therefore somehow different from the reality of the asymmetrical structure. Aoki (1993), as member of Hidaka's team of Hagia Sophia Surveying Project, made a tentative one-point measurement using micro-tremor sensors. The research was followed by Hanazato (2001), who implemented the simultaneous micro-tremor observation using 5 channels, and obtained the natural

[1)] Graduate Student
[2)] Professor
[3)] Professor

frequency as well as estimations of the mode shapes of each main pier.\

However, the one-point micro-tremor measurement, while making it possible to grasp the natural frequency of one spot, does not clarify the general characteristic behavior of the structure as a whole. Even with multi-point simultaneous measurements the vibration mode of only one axis can be acquired.

This paper aims to point out the structural dynamic behavior of Hagia Sophia as a whole, which is somehow different from the characteristics given in those relevant papers published in the past. Micro-tremor sensors were thus set-up in 28 points (84 channels) and the 3-D mode shapes of the primary structure as a whole are shown as follows.

2. MESUREMENT METHOD

The vibration properties can be obtained by several methods, the compulsory vibration examination, the free vibration examination, the earthquake observation and the micro-tremor observation. The micro-tremor observation method was adopted in this study because it is non-destructive. The research was done from Aug. 31th 2011 to Sep. 9th 2011. In order to measure the response acceleration, the sampling frequency was set up at $f = 40\text{Hz}$, and the measuring term was 3600 sec. For making all channels of micro-tremor sensors turn to the same direction, the positive directions of the x-axes faced the northern side of the cathedral and the z-axes faced upwards. The primary structure of Hagia Sophia is composed of main arch, main pier and secondary pier. Therefore micro-tremor sensors were set up at the top of the main arches, and the ground level, the gallery level and the second cornice level of the main and secondary piers all of 28 points as shown Fig. 1.

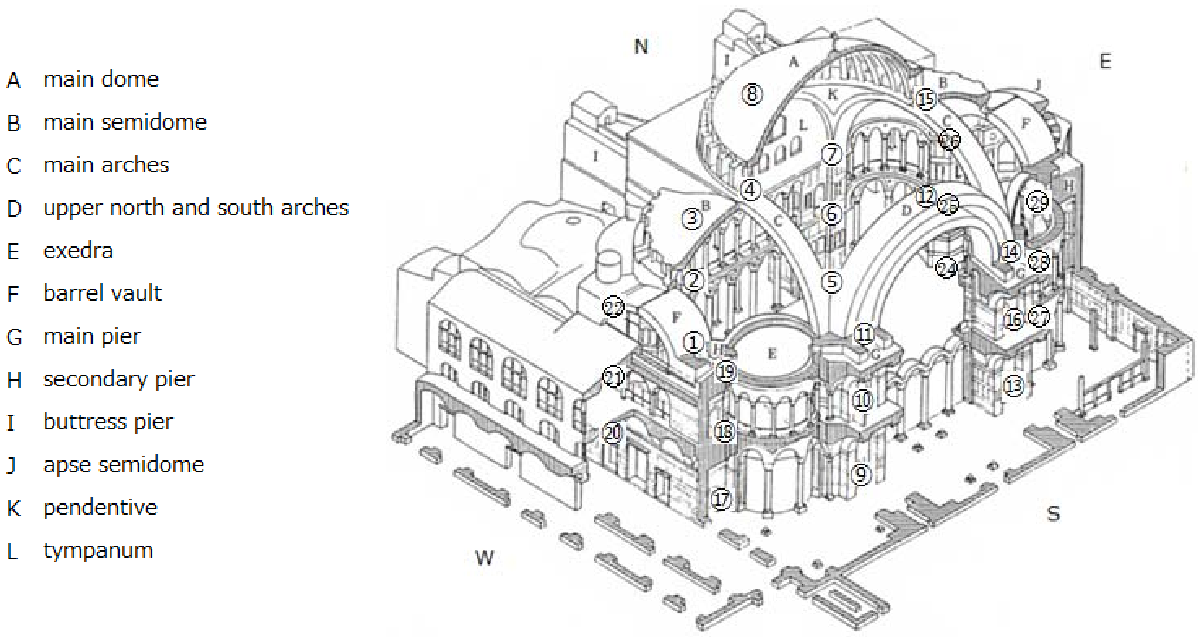

Fig. 1 Hagia Sophia and the observation points

3. ANALYTICAL METHOD

In this study, Frequency Domain Decomposition was adapted to show the total vibration characteristics of the cathedral (ARTeMIS, Structural Vibration Solutions). This method includes up to three frequency domain modal analysis techniques derived from the patented FDD (a type of frequency domain analysis) technology utilizing the singular value decomposition of the estimated spectral densities of the measurement response. The distances between observed points of the cathedral were measured on CAD based on a drawing of Van Nice (1965).

When there was proximity natural frequency, the effective technique is derived from FDD. The spectral response matrix was obtained by the observation made by singular value decomposition at each frequency, and the damping is estimated by the natural frequency and eigenvectors. It is possible to extract a single mode component even if there are the proximity modes. This methodology was developed by Brinker (2000).

4. ANALYTICAL RESULTS

The structural dynamic characteristics of Hagia Sophia were estimated based upon micro-tremor records. Peaks corresponding to the natural frequencies were acquired. The examples of these natural frequencies and mode shapes will be given as below in Figs. 2-6.

(1) Mode shape ($f = 1.722\mathrm{Hz}$, Fig. 2)
Large amplitude existing in the top of the north main arch and southeast second cornice could be confirmed. The north main arch has notable dynamic behavior vibrating up northwards and down southwards in particular.

(2) Mode shape ($f = 2.115\mathrm{Hz}$, Fig. 3)
The top of the north main arch shows the same behavior as the former one. Concerning the main piers, the ground level of the southeast main pier has large amplitude.

(3) Mode shape ($f = 4.029\mathrm{Hz}$, Fig. 4)
The top of the north main arch holding the remarkable behavior resembles the behavior of those with lower frequencies.

(4) Mode shape ($f = 5.392\mathrm{Hz}$, Fig. 5)
The gallery level of the northwest main pier vibrates with large amplitude.

(5) Mode shape ($f = 19.91\mathrm{Hz}$, Fig. 6)
The primary structure of the cathedral vibrates to the east and west axis as a whole. The stability of Hagia Sophia towards the east-west axis was smaller than towards the north-south axis as was stated in the preceding research. Thus, the buttress piers prevent the displacement caused by the thrust force from the north-south axis.

Of particular importance were the inconsistent vibrations shown in the uppermost part of the north main arch bordering the semi-circle of the northern tympanum. The results of our measurements may contribute to the clarification of the difference between the northern and the southern arch, which are supposed to be symmetrical on a structural level.

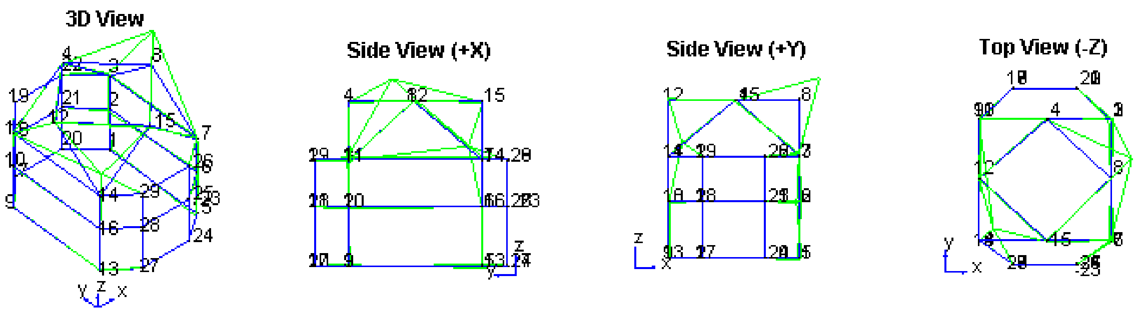

Fig. 2 Mode shape ($f = 1.722\mathrm{Hz}$, Amplitude 28%, Phase Angle 90’)

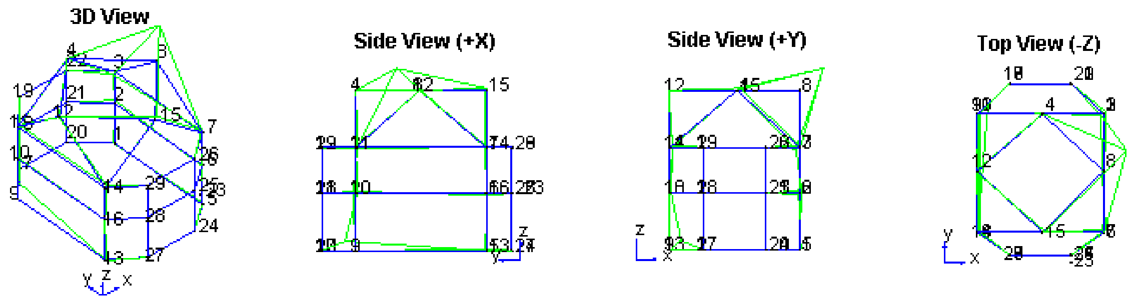

Fig. 3 Mode shape ($f = 2.115\mathrm{Hz}$, Amplitude 28%, Phase Angle 90’)

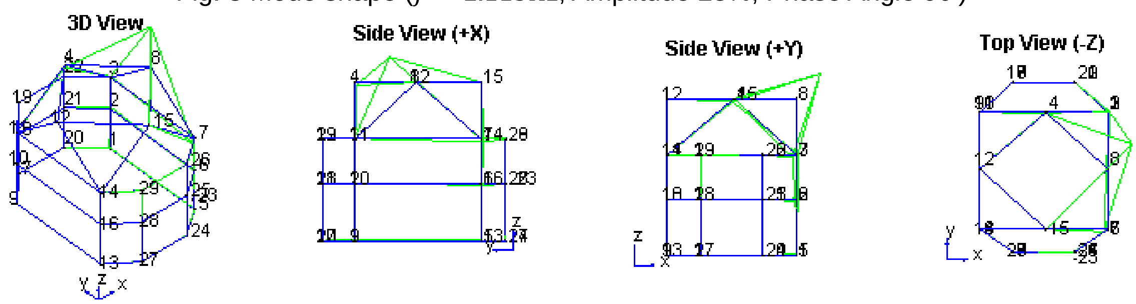

Fig. 4 Mode shape ($f = 4.029\mathrm{Hz}$, Amplitude 28%, Phase Angle 90’)

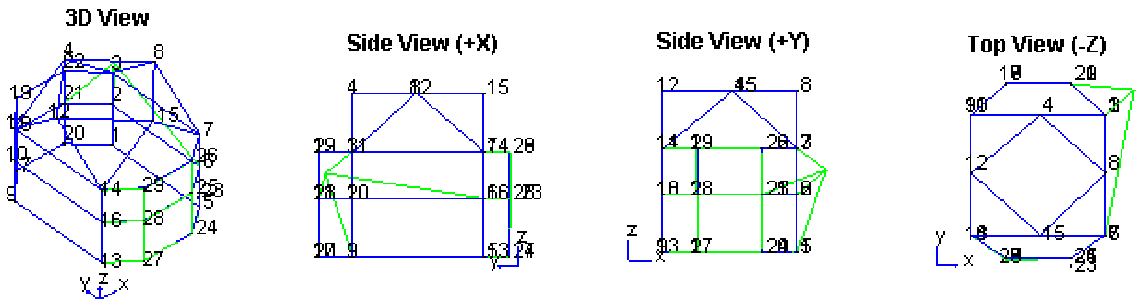

Fig. 5 Mode shape ($f = 5.392\mathrm{Hz}$, Amplitude 28%, Phase Angle 90’)

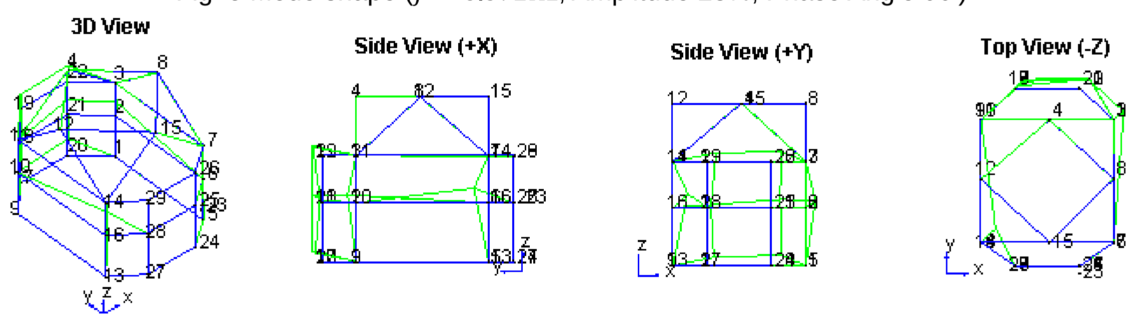

Fig. 6 Mode shape ($f = 19.91\mathrm{Hz}$, Amplitude 28%, Phase Angle 90’)

5. CONCLUSIONS

In this paper, the structural dynamic behaviors of the primary structure of Hagia Sophia as a whole were clarified using FDD based on the micro-tremor records. With some clarification of the fact so far unknown, the conclusions of this study are given below.

(1) The 3-D vibration mode shapes and natural frequencies of the primary structure of Hagia Sophia are estimated using FDD.
(2) The top of the north main arch has a remarkable mode shape vibrating up northwards and down southwards, with large amplitude of vibration.
(3) The results of our measurements may contribute to the clarification of the difference between northern and southern arch, which at first sight seem to be symmetrical.
(4) The stability of Hagia Sophia in the east-west axis was smaller than that in the north-south axis as was reported previously.
(5) The buttress piers prevent the displacement caused by the thrust force from the north-south axis.
(6) It is possible to extract a single mode component even if there are the proximity modes.

6. ACKNOWLEDGMENT

This research work was done under the supports of Grants-in-Aid for Scientific Research, Japan Ministry of Education, Culture, Sports, Science and Technology. (No.21226014)

REFERENCES

Mainstone, R.J. (1988), “Hagia Sophia: Architecture, Structure and Liturgy of Justinian’s Great Church”, London, Thames and Hudson.

Cakmak, A.S., Taylor, R.M. and Durukal, E. (2009), “The structural configuration of the first dome of Justinian’s Hagia Sophia (A.D. 537-558): An investigation based on structural and literary analysis”, *Proceedings of Soil Dynamics and Earthquake Engineering*, 29.

Ozkul, T.A. and Kuribayashi, E. (2007), “Structural characteristics of Hagia Sohia: Ⅱ-A finite element formulation for Dynamic analysis”, *Building and Environment*, 42.

Aoki, T., Ishikawa, K. and Kato, S. (1993), “Micro-tremor Measurement of Hagia Sophia”, *Architectural Institute of Japan*.

Hanazato, T. and Takeyama, H. (2001), “Study on Structural Stability of Hagia Sophia”, *Hagia Sophia Surveying Project Conference*.

Van Nice, R.L. (1965), “Saint Sophia in Istanbul: an architectural survey”, Washington, Dumbarton Oaks.

Brinker, R., Zhang, L. and Andersen, P. (2000), “Modal identification from ambient responses using frequency domain decomposition”, *Proceeding of IMAC*, 18.