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Development of a particle difference scheme for dynamic crack propagation problem

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ABSTRACT

This paper presents a novel Particle Difference scheme for solving dynamic crack propagation problems. The Particle Difference scheme directly discretizes the governing partial differential equations and yields a strong formulation. The scheme is based on nodal computation using the Taylor polynomial expanded by the Moving Least Squares method so that it no longer relies on the mesh or grid structure. Topology change due to crack propagation induced by a dynamic impact loading is easily modeled by involving very small modification of node arrangement. An efficient dynamic algorithm is selectively adapted for the Particle Difference scheme and the visibility criterion and dynamic energy release rate evaluation are combined with the scheme for crack propagation modeling. Numerical examples thoroughly verified the robustness and effectiveness of the scheme.

1. INTRODUCTION

In numerical simulations for the dynamic crack propagation driven by an impact loading, there has been a cumbersome issue on geometrical modeling of topology change. For example, when using Finite Difference Method(FDM) or Finite Element Method(FEM), internal boundary like a crack ligament is generally fixed to grid or mesh structure and the evolution of the boundary increases complexity and difficulty in the management of numerical model. Grid modification or mesh reconstruction provokes considerable inconvenience in transient simulation whether it is partial or not. In order to circumvent this inconvenience, the XFEM(eXtended Finite Element Method; Menouillard, 2009) was applied to the simulation of dynamic crack propagation; however, numerical integration issue has apparently remains as a troublesome work to be resolved. On the other hand, the meshfree method like EFGM(Element Free Galerkin Method; Belytschko, 1994) that was developed to overcome mesh dependency has struggled with handling weak formulation in the topology change involving simulation despite the node-wise character of the approximation function.

The Particle Difference Method, which is built up with Taylor polynomial expanded

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by Moving Least Squares method using node-wise discretization, can sophisticatedly solve dynamic crack propagation problem involving topology change; it is based on the strong formulation so that it enables one to simulate the problem by using nodal computation only. Thus, crack growth phenomenon can be very effectively traced by the method. This study presents the Particle Difference scheme for simulating dynamic crack propagation induced by an impact loading.

2. DYNAMIC CRACK ANALYSIS

2.1 Particle Difference Scheme for Dynamic Problem

When considering linear elastic fracture mechanics with dynamic response under small deformation assumption, the governing equations are given by the transient form of Navier's equation, natural boundary condition and essential boundary condition taking the form, respectively (See Lee and Yoon, 2004)

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) = \rho\mathbf{a} \quad \text{in } \Omega \tag{1}$$

$$\mu\mathbf{n}\cdot\left(\nabla\mathbf{u}+\nabla\mathbf{u}^T\right) + \lambda\mathbf{n}\cdot\mathbf{1}(\nabla\cdot\mathbf{u}) = \bar{\mathbf{t}} \quad \text{on } \Gamma_t \tag{2}$$

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_u \tag{3}$$

where $\mathbf{u}$ denotes the displacement which is the function of time and space, ρ the material density, $\mathbf{a}$ the acceleration, $\mathbf{n}$ the unit normal to the natural boundary, $\bar{\mathbf{t}}$ the prescribed traction and $\bar{\mathbf{u}}$ the prescribed displacement; $\mathbf{1} = \delta_{ij}\mathbf{e}_i \otimes \mathbf{e}_j$ is a second order identity tensor, λ and μ the Lamé constants and $\nabla^2\left(=\nabla\cdot\nabla\right)$ the Laplace operator.

In the Particle Difference Scheme, the derivative approximation is derived by the combination of the Taylor expansion and the Moving Least Squares method. The aforementioned strong forms can be directly discretized using the derivative approximation as the partial differential equations are numerically differentiated in the framework of FDM. The strong formulation at an arbitrary node $\mathbf{x}_J$ is written in a matrix form as following

$$\left(\underset{I=1}{\overset{N}{A}}\mathbf{K}_J\right)\cdot\mathbf{u}_J = \left(\mathbf{F}_J\right) \tag{4}$$

where $\mathbf{u}_J = \left(u_{xJ}, u_{yJ}\right)^T$ is the nodal solution vector for node $\mathbf{x}_J$ and $\mathbf{F}_J = \left(F_{xJ}, F_{yJ}\right)^T$ the force vector. $\mathbf{K}_J$ is the 2×2 stiffness matrix consisting of the stiffness components for the neighbor nodes included in the approximation construction and A denotes the assembly operator. $\mathbf{F}_J$ and $\mathbf{K}_J$ take different form according to the governing equation considered. For time integration of Eq. (1), the Newmark method is employed.

2.2 Dynamic Crack Propagation Modeling

The visibility criterion(Fleming *et al*., 1997) is introduced to model crack discontinuity, which discards the nodes placed at the opposite side of the crack when the crack cuts through the domain of influence of a sampling point. In order to check whether the crack will grow or not, dynamic stress intensity factor(Freund, 1990) is evaluated and it is computed from the dynamic energy release rate which takes the form

$$g = \frac{1-\upsilon^2}{E}\left(A_I\left(v_c\right)K_I^2 + A_{II}\left(v_c\right)K_{II}^2\right) \tag{5}$$

where E is the Young's modulus, ν the Poisson's ratio and v_c the crack tip velocity. $A_I\left(v_c\right)$ and $A_{II}\left(v_c\right)$ are wave related parameters and K_I and K_{II} are the dynamic stress intensity factors for mode I and II, respectively. The energy release rate is computed by integration of the momentum equation over J integral domain. From Eq. (5), equivalent stress intensity factor is calculated and is compared with the dynamic fracture toughness of the given material in order to check whether the crack will grow or not. The mode I equivalent stress intensity factor is calculated by

$$K_{Ieq}\left(v_c\right) = K_I\Sigma_h^I\left(\theta, v_c\right) + K_{II}\Sigma_h^{II}\left(\theta, v_c\right) \tag{6}$$

where θ is crack growth angle. The mode II equivalent stress intensity factor can be obtained in the same way. Actually, new traction-free surface needs to be created to model growing crack. The Particle Difference scheme easily carries out this operation with simple node movement and addition.

3. NUMERICAL EXAMPLE

A notched plate subjected to an impact loading is simulated to verify the performance of the developed scheme for 2-D dynamic crack problem.(See Fig. 1(left)) The constant velocity impact loading(V_0) is applied along the lower edge of left side of the plate and traction-free boundaries are imposed along the remaining part of boundary to asymptotically model the infinite domain. In the simulation, $\rho = 7833\,kg/m^3$, $E = 200\times10^9\,N/m^2$ and $\nu = 0.25$ are assumed. 240(40×60) node model is used and 80 step computations are performed with $\Delta t = 6.5\times10^{-6}\sec$. Also, the dynamic stress intensity factor is computed using $1m\times1m$ J-domain around the crack tip. In Fig. 1(right), the numerical result for the dynamic stress intensity factor is compared with the closed form solution and that of EFGM; time is normalized by wave speed c_d and the initial crack length a_0. Very good agreement is easily found between them.

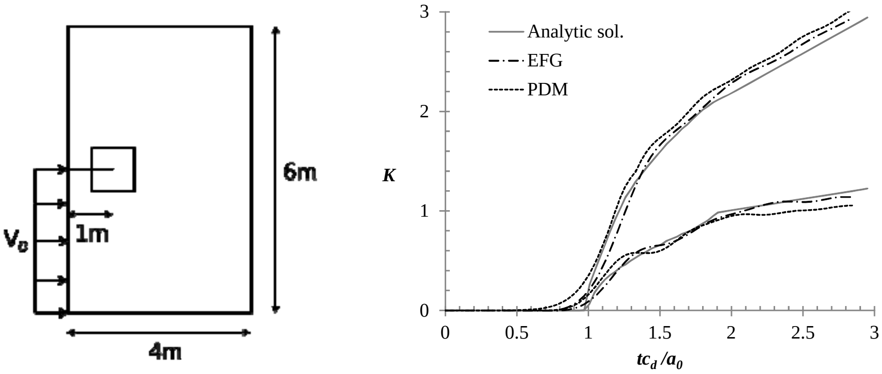

Fig. 1 Configuration and numerical result: Problem illustration(left); dynamic stress intensity factors for the stationary edge-crack problem(right)

4. CONCLUSIONS

In this study, a strongly formulated particle difference scheme is presented for the simulation of dynamic crack propagation. The scheme is completely mesh-free or grid-free such that only nodal computation is involved without numerical quadrature. Topology change resulting from the evolution of internal boundary due to fracture of a solid body is successfully modelled. The visibility criterion is employed for the description of crack discontinuity and the dynamic energy release rate is evaluated to determine crack growth direction as well as to check whether the crack will grow or not. The developed scheme is able to efficiently trace the transient crack propagation phenomenon with minimal modification of node arrangement. Numerical example reveals that the numerical scheme achieves very good accuracy and robustness. It is noteworthy that the particle difference scheme has strong merits in the simulation involving evolving boundary with topology change. The scheme is also expected to be straightforwardly applied to the dynamic problems with material nonlinearity.

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