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Vibration based health monitoring of joints in a continuous beam structure

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ABSTRACT

In the construction of civil engineering structures, two or more beams are often rigidly connected to make it continuous over multiple supports in order to increase the structural integrity. These rigid joints are often designed with bolts, rivets and welding. The action of in-service loading and environmental effects make these joints semi-rigid, which ultimately reduces the structural reliability. In this study, a theoretical model based on Euler-Bernoulli beam theory of a two span continuous beam with semi-rigid joints is considered. The beam is modeled using 1-D beam element in which two zero length rotational springs are considered at both the ends. The damage is simulated by reducing the stiffness of rotational springs. The elemental mass and stiffness matrices are dependent on the stiffness of the rotational springs, which contribute to the changes of the matrices due to semi-rigidity of the joints. Two damage detection algorithms, namely modal curvature (MC) and modal strain energy (MSE), are applied to identify the damaged joint. A damage index is defined using the baseline and the damaged mode shape. Various damage cases with different levels of noise are studied. It is found that damage indices are more pronounced near or at the location of damage.

1. INTRODUCTION

The steel structures are often designed for the construction of industrial buildings, offshore structures and bridges due to higher reliability and lesser construction time. In these structures, the components like beam and columns are modeled with bolts or rivets or by welding. Relaxations, yielding of bolts, thread stripping, improper welding and the service related loads or an environmental effect degrades the strength of the joints over time. Therefore, to ensure life safety and economy, non-destructive evaluation (NDE) of the structure is required throughout its life span by deploying contact or non-contact type of sensors network. This NDE can be classified into local and global identification technique. For the health monitoring of small structure, local identification technique can be applied but, for large and complicated structure, vibration based global health monitoring techniques are postulated which can assess the entire structure at once (Farrar and Doebling, 1997; Medhi *et al.*, 2008; Banerji and Chikermane, 2012).

At the very early stage of vibration based damage identification, shift of resonant

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frequency was popular one for identification of damage as the frequency is easy to obtain and required lesser number of sensors. But computational costs for identification, precise measurement made this technique obsolete (Adams *et al.* 1978, Kim and Stubbs 2003, Kim *et al.* 2007b, Sato 1983, Stubbs and Osegueda 1990). Mode shape methods are based on the fact that mode shape is function of the physical properties of the structure. Therefore, changes in the physical properties will cause detectable changes in the mode shape. Plenty of studies have been found using mode shape on beam type of structure. Modal assurance criterion (MAC), Coordinate modal assurance criterion (COMAC) and absolute difference between undamaged and damaged modal curvature was developed for identification of existence of damage and location of damage (West 1984, Yuen 1985, Yao et al. 1990, Pandey *et al.* 1991 and Fox 1992). Laplacian operator was applied on mode shape of a damaged beam for identification of location. For lower level of damage, a cubic spline was fitted to the Laplacian and from the difference between spline and Laplacian location of damage was determined. The technique was also validated experimentally. This method works well when mode shapes are extracted from fundamental frequency (Ratcliff 1997 and Kim and Stubbs 2002). Shi *et a*l. (1998) formulated elemental modal strain energy change ratio for identification of damage. The technique was applied on a simulated fixed-fixed beam model, where the damage was introduced by reducing the cross section over an element. It was concluded that this kind of damage has no effect on the strain energy change ratio (MSECR) of the elements of columns in the frame. The damage can be identified from the maximum change of MSECR of the element of beams attached to the joint (Shi et al., 1998 and Shi et al., 2002).

FRF curvature based identification was proposed by Sampaio et al (1999). They applied the technique on I-40 bridge data and compared with modal curvature and found that this technique is not noise sensitive and work well for lower frequency range. Modern signal processing tools like wavelet transform, EMD and Hilbert-Huang transform have been used widely to identify the damage parameters. Multi-resolution property of this technique makes it popular one. Wavelet transform is performed on spatial signal like mode shape, static displacement to determine the location of the damage. Again damage time instant is determined from the variation of wavelet coefficient over time (Rajasekaran and Varghese, 2005; Wang et al., 1999).

In model based identification, a numerical model is considered as the baseline structure and updates the model until it represents the experimental model. Conventional optimization techniques require initial values of parameters near the actual values (Zapico *et al.* 2006). Therefore, population based search algorithm in the evolutionary framework are deployed in the field of SHM and its intelligent search technique is furnishing promising results. Several studies have been found on the identification of damage on a beam type of structure. Objective functions are developed using residual forces, natural frequencies, mode shapes etc. from multiple modes. Damage is simulated either by providing a rotational spring or by reducing moment of inertia of an element (Frishwell *et al.* 1998, Rao *et al.* 2004, Baghmisheh *et al.* 2008 and Meruane and Heylen 2010). Perera *et al.* 2007) applied multi-objective genetic algorithm to determine the damage. The effectiveness is verified by simulated beam with noise and also by using experimental data. As an extension of the previous study, Perera *et al.* (2009) takes into account the modeling error. Depending on the complexity of the problem, numbers of variable increases which makes the problem computationally inefficient. Few studies have been found on the search space reduction technique using some sampling technique. Again, the results are improved by applying gradient search (Perry *et al*. 2006, Zhang *et al*. 2010 and Zhang *et al*. 2010).

Force displacement relationship for the displacement based structural analysis considers fixed-fixed boundary condition. Insufficient clamping force, joint relaxation, yielding of bolts, thread stripping and environmental variability reduces the fixity level at boundary. Monforton and Wu (1963) developed a modified force displacement relationship in terms of elemental stiffness matrix for elastically restrained boundary conditions. Using the conjugate beam method of structural analysis they have developed a force displacement relationship in terms of a matrix form. Monforton and Wu's formulation can only provide the joint or the member end displacement. Chan and Ho (1994) proposed a numerical method for linear and a non-linear dynamic analysis of frame with semi-rigid connection. They have derived a shape function for semi-rigid boundary condition and used it to formulate elemental matrices. The semi-rigidity of the boundary is provided by rotational springs. Chui and Chan (1997) studied vibration and deflection characteristics of semi-rigid jointed frames based on Chan and Hos's (1994) formulation. The objective of their study is to inspect serviceability and deflection under vibration. They have experimentally determined the rotational spring stiffness.
From the brief review of past research, it appears that most of the techniques can be applied on the beam type of structure to identify the existence, location and quantity of damage. Surprisingly, very limited amount of literature is available addressing problems involving health monitoring of joints. In this paper, a theoretical model is described for vibration based detection and characterization of semi-rigid joints in a continuous beam. The elements of the frame model consist of two zero length rotational springs at their two ends to represent degree of semi-rigidity. The vibration model of the beam accounts for the rotational stiffness of the springs in the stiffness matrix and consistent mass matrix. Damage detection algorithms based on MC and MSE are used for health monitoring of joints. Using the initial calculations performed on a structure with rigid joints as baseline, the damage indices are evaluated at several control points from the comparison of the modal response of the monitored structure with semi-rigid joints.

2. FORMULATION OF ELEMENTAL MATRICES

In order to monitor the joints, a vibrational analysis which allows joints flexibility is adopted. First mass and stiffness matrices are derived neglecting all kinds of non-linearity. In figure 1(a), the deflected shape of an element is shown. The element consists of two springs at the end to consider the end flexibility. Therefore, a relative rotation will take place in the inner and outer side of the spring. In inner side, the rotations are $L\theta_1$, $R\theta_1$ and in outer side, these are $L\theta_2$, $R\theta_2$ respectively. According to Monforton and Wu (1963) –

$$\frac{RM}{Rr_c} = R\theta_2 - R\theta_1 \quad \text{and} \quad \frac{LM}{Lr_c} = L\theta_2 - L\theta_1 \tag{1}$$

In the above equation, LM , RM , Lr_c and Rr_c are the applied moment and rotational spring constants at left and right side of the element respectively. The displacement profile over an element is derived as per Chen and Hu's (1994) formulation. \

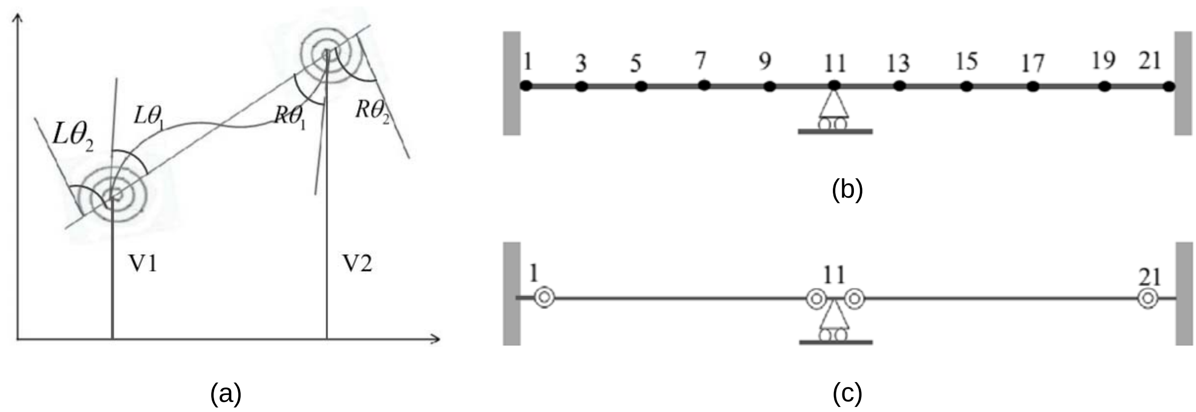


Fig. 1(a) Deformation of an element due to end displacement; (b) two span continuous beam model (make a roller at the intermediate span); (c) representations of joints using rotational springs

$$y=\begin{bmatrix} P_1^2P_2L & P_1P_2^2L \end{bmatrix}\left(-\begin{bmatrix} \frac{4EI}{L}+Lr_C & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L}+Rr_C \end{bmatrix}^{-1} \begin{bmatrix} -Lr_C & 0 \\ 0 & -Rr_C \end{bmatrix} \begin{bmatrix} \frac{1}{L} & 1 & -\frac{1}{L} & 0 \\ \frac{1}{L} & 0 & -\frac{1}{L} & 1 \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} \right) + v_1P_1 + v_2P_2 \qquad (2)$$

In the above expression,

$$\left(1-\frac{x}{L}\right)=P_1 \quad , \quad \frac{x}{L}=P_2$$

where, L and x are the length and spatial distance of a point from the left node of an element respectively.

Stiffness and mass matrix can be determined by the standard procedure.

$$k=\int_0^L EI\left[N''(x)\right]^T\left[N''(x)\right]dx \qquad (3)$$

$$m=\int_0^L \overline{m}\left[N(x)\right]^T\left[N(x)\right]dx \qquad (4)$$

In the above expressions, $\overline{m}$, *E* and *I* are the distributed mass, young modulus and moment of inertia, respectively, over the element.

3. HEALTH MONITORING PROCEDURE

Mode shapes corresponding to the first natural frequency has been determined and two damage detection algorithms, namely modal curvature and modal strain energy are deployed to monitor the joints of the structures. The calculation of damage indices is described below.

3.1 Modal curvature

Modal curvature is calculated using the second order central difference scheme. The expression can be given as below [Pandey *et al.* (1991)] –

$$y_i^{"} = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} \tag{5}$$

where, y and h are the mode shape displacement and nodal distance respectively. Damage index is calculated from the square of the difference between the damaged and undamaged curvature.

$$DI = \left| y_d^{"} - y^{"} \right|^2 \tag{6}$$

3.2 Modal strain energy

Modal strain energy can be expressed as [Lin and Cheng *et al.* (2008)] –

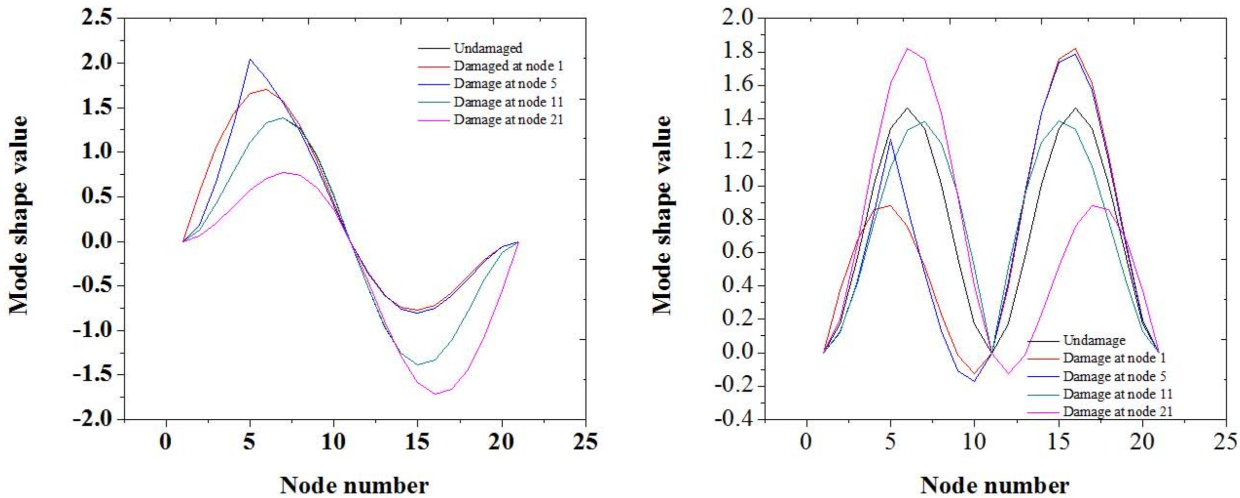
$$U = \int_0^L EI \left(\frac{d^2 y}{dx^2} \right)^2 dx \tag{7}$$

where, U, E, I and $d^2 y / dx^2$ in the above equation are the strain energy, Young's modulus, moment of inertia and curvature respectively. Damage index is calculated from the square of the difference between the damaged and undamaged nodal modal strain energy.

$$DI = \left| MSE_d - MSE \right|^2 \tag{8}$$

4. NUMERICAL EXAMPLES AND RESULTS

In order to examine the performance of the proposed methodology, a two span continuous beam model as shown in the figure 1 (b) has been considered. The beam has two fixed supports at the two ends and one simply support at the middle. The number of rotational springs at each joint depends on the number of members meeting at a joint as shown in Figure 1(c). For example, the joint involving the fixed support at each end is represented by one rotational spring, whereas, two rotational springs are required to represent the simply support at the middle. If a section of any one of the span is considered to be damaged, then two rotational springs shall be introduced to represent the damaged section. In that case, the damaged section will be termed as a joint. The beam is modeled using 2D beam element in MATLAB environment. The level matrices are developed using the shape function give in Eq. (3) and Eq. (4). The value of undamaged spring constant is considered as 1.36×10^7 N-m. To simulate the damage, any one of the rotational spring stiffness of a joint is reduced by 90%. Several damage locations including both the support joints (fixed and simply support), only one of the support joints (fixed or simply support) and a nodal point between two joints are studied. Modal analysis is carried out to find out the modal data. Synthetically generated zero-mean Gaussian noise has been randomly added to the simulated data. It is also considered that the undamaged and the damaged data are collected from the same level of noise. The corresponding noise level is 80db. The percentage level of noise is multiplied by the standard deviation of each sequence of mode shape and randomly added to the components of the sequence.



(a) (b)

Fig. 2 (a) 1st mode shape; (b) 2nd mode shape of the continuous beam for undamaged and different damage locations

Table 1 Damage details and corresponding plots

Damage case number	Joint Location	Node number	Corresponding plot
1	Fixed support	Node 1	Fig 3(a), 4(a)
2	Simply support	Node 11	Fig 3(b), 4(b)
3	Fixed support	Node 21	Fig 3(c), 4(c)
4	In the span	Node 5	Fig 3(d), 4(d)

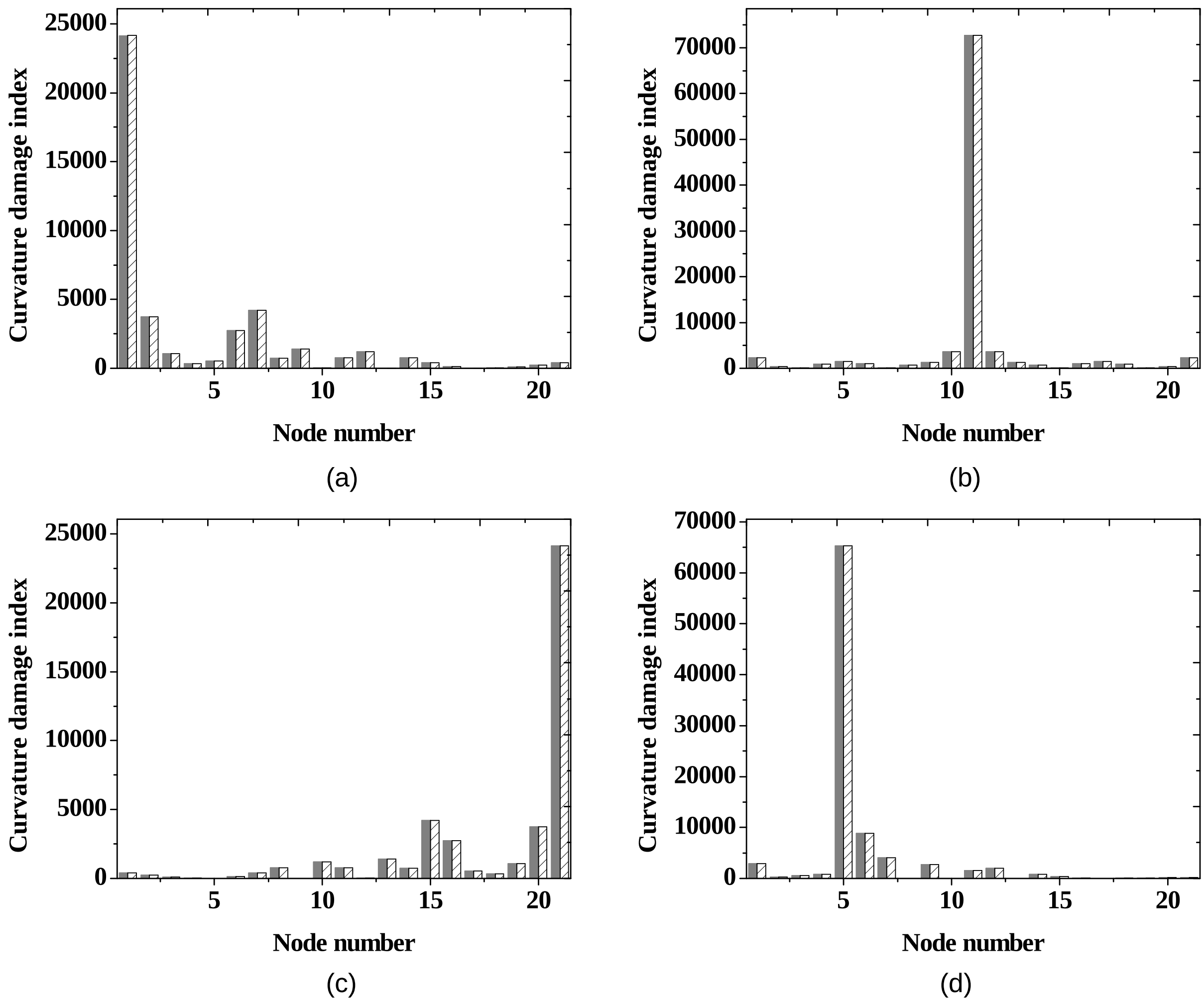


Fig. 3 Curvature damage index (without noise, with 80db noise): (a) For damage at node 1, (b) For damage at node 11, (c) For damage at node 21, (d) For damage at node 5

The second mode shape data are collected, and the MC and the MSE algorithms (Eq. 6 & 8) are applied to the data to identify the location of damage. The details of the damage cases studied and the corresponding results are reported in Table 1. Figures 3(a), 3(b), 3(c) and 3(d) are the modal curvature damage index plots for damage at node1, damage at node11, damage at node 21 and damage at node 5 respectively. The figures 4(a), 4(b), 4(c) and 4(d) are the modal strain energy based damage index plots for the same damage cases respectively. It is clear from the above figures that both the MC and MSE algorithms can identify the damage when data are collected from ideal conditions. It is also found that up to 80db of noise and that the noise has no significant effect on the damage indices and the maximum values of damage indices lie closer to the location of damage.

From the different damage cases, it is clear that the proposed methodology can identify the location of damage while damage may exist in support, in joints or in span. MC damage index identifies the exact location or the node, where the damage is located. MSE based DI, on the other hand, identifies the element in which rotational spring stiffnes is attached. In fact, the higher value of DI is found in the element sharing the node of the damaged element near the rotational spring. The noise test confirms that the proposed methodology can be applied for health monitoring of beam structures.

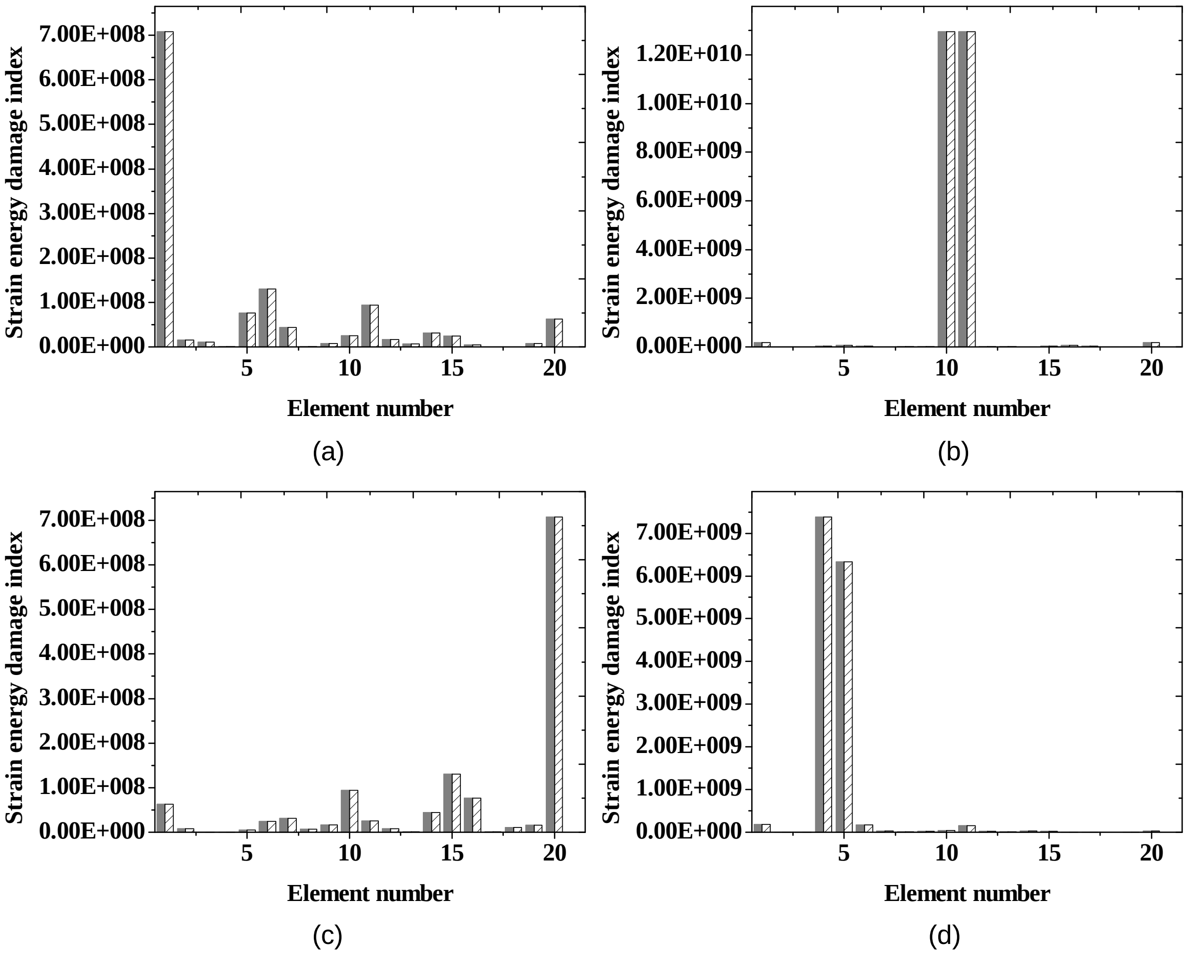


Fig. 4 Modal strain energy damage index (without noise, with 80db noise): (a) For damage at node 1, (b) For damage at node 11, (c) For damage at node 21, (d) For damage node 5

5. CONCLUSIONS

In this present study, a health monitoring methodology is presented to identify the location of damaged joint in a continuous beam. The beam is modeled with 1-d beam elements and elastic boundary conditions. In the numerical study, joint damage is simulated by reducing the rotational spring stiffness. To localize the damaged joint, modal curvature and modal strain energy methods are applied. In general, it has been observed that the methodology can identify the damage with reasonable noisy data of

80dB. While modal curvature and modal strain energy algorithms show some false alarms away from the damaged nodes and elements, respectively, the maximum value of DI is always found in the damaged node and the element. Therefore, this technique can be applied to the real life structure in which bolts or rivets may be loosened at the beams.

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