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Optimum structural configuration of irregular buildings
1. Elastic systems

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ABSTRACT

A modified procedure is presented for estimating frequencies and the peak values of base resultant forces of eccentric irregular buildings due to earthquake ground motions characterized by response spectra. This procedure retains the simplicity of the methodology presented by the first of the authors in earlier papers, but it presents higher accuracy in multi-story buildings composed by very dissimilar types of bents, which are usually defined as irregular structures. As a result, the first mode center of rigidity (m_1-CR) is determined with superior accuracy and this allows the practicing engineer to form a structural configuration which will sustain minimum rotational response, simply by allocating the resisting elements in such a way that this point lies close to the axis passing through the centers of the floor masses. It is demonstrated in the companion paper, that inelastic asymmetric building systems with a similar structural arrangement are also very effective during an earthquake, since they present an almost translational response. The accuracy of the proposed modified procedure is illustrated in mixed-bent-type eight-story structures, which are characterized by Eurocode (EC8-2004) as irregular structures, and comparisons are made with the accurate results obtained by response spectrum analyses using the SAP2000 computer program.

1. INTRODUCTION

It has been shown that the response of asymmetric buildings having resisting elements with stiffness matrices which are proportional to each other (proportionate buildings), and in which the centers of mass of all floors lie on the same vertical line, can be obtained by determining (i) the response of the corresponding uncoupled multi-story structure and, (ii) for each mode of vibration of the latter structure, by analyzing an associated torsionally coupled single story system (Kan and Chopra 1977a, Hejal and Chopra 1989a, Athanatopoulou et al 2006). The same analysis also holds for shear type buildings with different static eccentricities at the various floor levels (Kan and Chopra 1977b). Recently, this analysis was extended by the first of the authors to non-

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proportional buildings, by introducing the concept of the modal stiffnesses of the bents which provide the lateral stiffness of a given structure. It is shown (Georgoussis 2009, 2012) that the peak elastic response of medium height buildings can be derived by analyzing two equivalent single-story modal systems, each of which has a mass equal to the *k*-mode effective mass, M_k^* (*k*=1,2) of the uncoupled multi-story structure, and is supported by elements with stiffnesses equal to the product of M_k^* with the first mode (when *k*=1) or second mode (when *k*=2) squared frequencies of the corresponding real bents of the assumed multi-story structure. It is worth noting here that when the fundamental period of the uncoupled structure is in the acceleration sensitive region (that is, in the flat part of a typical response spectrum), the first mode (*k*=1) equivalent single story system is sufficient to provide reasonable estimates of the peak values of base resultant forces. The stiffness centre of this modal system constitutes the first mode centre of rigidity, m_1-CR (Georgoussis 2010) and when it lies close to the axis passing through the centers of floor masses, the rotational response sustained by the an elastic asymmetric building system is minimum. This response underlines the significance of the accurate evaluation of the location of m_1-CR in elastic systems and also, as it is demonstrated in the companion paper, in building systems which are expected to be deformed well into the inelastic region during a strong ground motion.
The concept of the aforementioned method is based on the potential of Rayleigh's quotients. These quotients, which traditionally are used to determine a close estimate of the first mode frequency of a given structure when an approximate first mode shape is used, are also useful in the case of building structures, which belong to the same family of shear-flexure cantilever systems and have mode shapes of the same form. For example, the first mode deflected shape presents one point of zero displacement (the base), the second mode shape presents two points of zero displacement, etc. In other words, in building structures, which in the general case may be envisaged as cantilever systems, the Rayleigh method can be used to determine the second mode frequency from an approximation of the second mode shape.
The objective of the present work is to improve the accuracy of the aforesaid methodology (Georgoussis 2011, 2012) by expressing the stiffnesses of the elements of the equivalent single-story modal systems (of either mode, *k*=1 or 2) by the product of the first mode (when *k*=1) or second mode (when *k*=2) squared frequencies of the actual bents with the corresponding effective modal masses of these bents and not with the effective modal mass of the uncoupled structure in the direction of the ground motion. The rest of the analysis remains the same, but this modified procedure is presenting results of superior accuracy when they are compared with the data obtained by response spectrum analyses using the SAP2000 computer program. This is demonstrated in structural systems with dissimilar bents, which are classified by EC8-2004 as irregular buildings.

2. ANALYSIS PROCEDURE

Consider the symmetrical plan of a uniform multistory building, as shown in Fig. 1. Although different types of bents are providing the required lateral resistance (*i*-bents are aligned along the x-direction and *j*-bents along the y-direction), the center of mass

(CM) coincides with the center of resistance of the structural system, since the resisting elements are pairs of bents aligned in two orthogonal directions, symmetrically to CM.

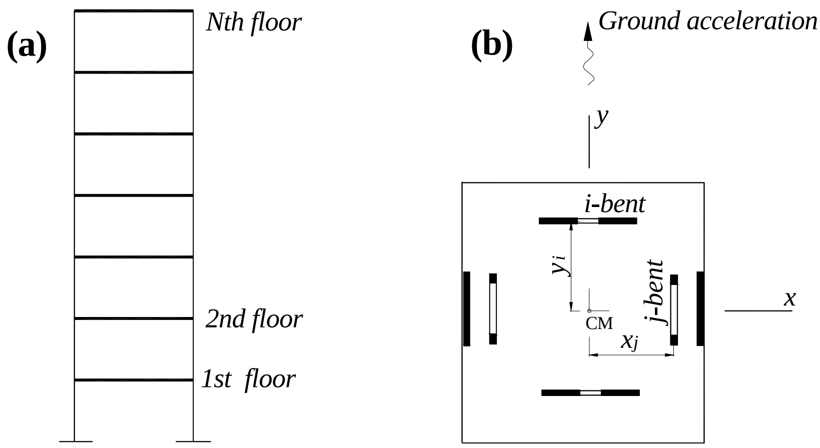

Fig. 1 (a) Elevation of a uniform multi-story building with; (b) symmetrical floor plan

Assuming that the example structure is subjected to a purely translational ground motion along the y-direction, essential dynamic quantities required for a structural design (e.g. peak base shears) may be determined from a combination of peak modal responses. Such a procedure requires the evaluation of the frequencies ω_{yk} and the effective masses M^*_{yk} (k=1,2,…) of single-degree-of-freedom (SDOF) modal systems. As an example, the first mode SDOF system of the assumed building, together with its corresponding modal shape, is shown in Figs. 2(a) and (b).

It is well known that the stiffness of the k-mode SDOF system is equal to

$$k^*_{yk} = \omega^2_{yk} M^*_{yk} \tag{1}$$

and an approximate estimate of its value can be obtained by taking into account that the stiffness $\mathbf{K}_y$ of the example structure (in a matrix form) in the assumed direction is equal to

$$\mathbf{K}_y = \Sigma \mathbf{K}_j \tag{2}$$

where $\mathbf{K}_j$ is the stiffness matrix of the j-bent. Denoting with ω_{jk} the k-mode frequency of the subsystem which has the same mass as the actual structure but its lateral stiffness depends entirely on the j-bent, a reasonable estimate of the k-mode frequency of the complete structure, ω_{yk}, may be determined by the formula (Georgoussis 2009)

$$\omega^2_{yk} \approx \Sigma \omega^2_{jk} \tag{3}$$

For the first mode of vibration, the expression above represents Southwell's formula (Newmark and Rosenblueth, 1971; Jacobsen and Ayre, 1958) which provides a close lower bound of the fundamental frequency of the real structure.

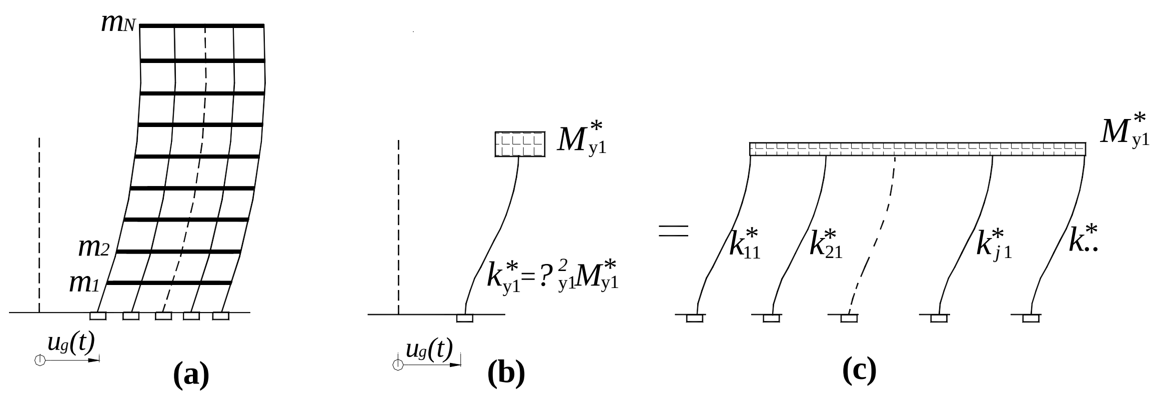

Fig. 2 (a) Typical first mode deformation profile of a multi-story building; (b) the first mode single-story system with the modal stiffness; (c) the contribution of each bent to the modal stiffness

Therefore, the *k*-mode stiffness of Eq.(1) may be approximated by the expression

$$k^*_{yk} = \Sigma(\omega^2_{jk} M^*_{yk}) = \Sigma k^*_{jk} \tag{4}$$

That is, the modal stiffness of the system of Fig. 2(b) can be considered as the sum of the modal stiffnesses of the *j*-bents (Fig. 2(c)), each of which is assumed to be equal to

$$k^*_{jk} = \omega^2_{jk} M^*_{yk} \quad (j=1,2,\ldots) \tag{5}$$

The formula above represents the basis of the simple procedure presented in previous papers (Georgoussis 2009, 2012), for the analysis of eccentric non-proportionate buildings under ground excitations. At present, a modified expression is proposed for the evaluation of the *k*-mode stiffness of the *j*-bent (subsystem), according to the following considerations:

The *k*-mode response of the aforementioned subsystem, which has the same mass as the actual structure but its lateral stiffness depends entirely on the *j*-bent (as shown in Fig. 3(a) for the first mode of vibration), can be derived from a SDOF modal oscillator, which has a stiffness given by

$$k^*_{jk} = \omega^2_{jk} M^*_{jk} \quad (j=1,2,\ldots) \tag{6}$$

where M^*_{jk} is now the *k*-mode effective mass of the particular *j*-subsystem, which is, in the general case, different from M^*_{yk}. For the first mode of vibration (*k*=1), this SDOF modal oscillator is shown in Fig. 3(b).

Since the stiffness of the real structure is given by the sum of the stiffnesses of the *j*-bents (*j*=1,2,...), as is indicated by Eq. (2), it is appropriate to assume that the *k*-mode stiffness, k^*_{yk} , of the corresponding SDOF modal system of the complete structure (shown in Fig. 2(a) for the first mode of vibration), which is given by Eq. (1)), is equal to the sum of the modal stiffnesses of Eq. (6), i.e.

$$k^*_{yk} = \Sigma k^*_{jk} = \Sigma(\omega^2_{jk} M^*_{jk}) = \Sigma(\omega^2_{jk} \frac{M^*_{jk}}{M^*_{yk}} M^*_{yk}) = \Sigma(\overline{\omega}^2_{jk} M^*_{yk}) \text{ , } (k=1,2,..) \quad (7)$$

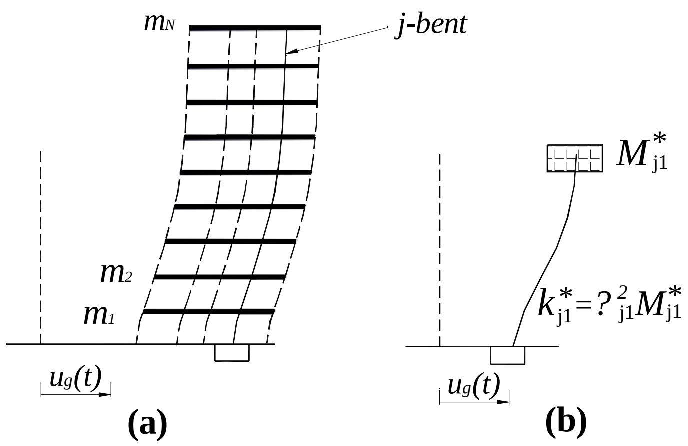

Fig. 3 (a) First mode displacement profile of the multi-story subsystem which has a lateral stiffness depending on the j-bent; (b) the corresponding SDOF modal system

This is an expression similar to Eq. (4), but it takes into account the effect of the mode shape of the particular bent on the corresponding effective modal mass. The sum of the modal stiffnesses of the various bents, given by Eq. (6), is approximating the modal stiffness of the complete structure k^*_{yk} more accurately than the sum of the Eqs. (5), and this approach makes the approximate method presented by the first of the authors in earlier papers more accurate. As shown in the next section, results (periods and base resultant forces) of higher accuracy are derived when the 'effective' frequencies, $\overline{\omega}_{jk} = \omega_{jk}\sqrt{M^*_{jk} / M^*_{yk}}$, instead of ω_{jk} (j=1,2,..), are used in the approximate method.

More details of how the modal shape of a cantilever system affects the effective modal mass M^*_{jk} are provided in Georgoussis (2009). This is shown in a parametric form (by means of the method of the continuum approach), for the first three modes of vibration (*k*=1,2,3). As demonstrated, in flexural cantilevers (frames with beams of zero stiffness) the first mode effective mass is about 60% of the total mass and this value is increasing to more than 75% in the case of frames with stiff beams (shear type systems). For the next two modes of vibration, the effective modal masses are higher in flexural cantilevers (approximately 19% and 6.5% respectively for the second and third mode of vibration) and they are taking lower values in shear type systems (about 9% and 4%).

Having introduced the concept of the 'effective' frequency for each of the resisting bents, the modified procedure for estimating periods and peak values of base resultant forces of uniform buildings with simple asymmetry is implemented as described by Georgoussis (2009, 2012) and, in brief, by the following steps:

First, by expressing the modal stiffness of any *i*-bent in the x-direction in a similar manner, e.g.

$$k_{ik}^* = \omega_{ik}^2 M_{ik}^* \tag{8}$$

where ω_{ik}, M_{ik}^* are the *k*-mode frequency and effective mass respectively of the subsystem which has the same mass as the actual structure but its stiffness depends entirely on the *i*-bent. Second, by forming the undamped equation of motion of the *k*-mode single-story system, which has a mass equal to M_{yk}^* and it is supported by elements having stiffnesses given from Eqs. (6) and (8). In a coordination system with the origin at the center of mass (as shown in Fig. 4(a) for the example structure), this equation, for a uni-directional excitation along the y-direction, is as follows

$$\mathbf{M}_k^*\ddot{\mathbf{U}}_k + \mathbf{K}_k^*\mathbf{U}_k = -\mathbf{M}_k^*\mathbf{1}\ddot{u}_g \tag{9}$$

Where

$\mathbf{M}_k^* = M_{yk}^*\begin{bmatrix}1 & 0\\ 0 & r^2\end{bmatrix}$ the effective *k*-mode mass matrix

(*r*: the radius of gyration of the typical floor)

$\mathbf{U}_k = \langle u_k \quad \theta_k\rangle^{\mathrm{T}}$ the corresponding modal displacement vector at CM

$\mathbf{K}_k^* = \begin{bmatrix}k_y^* & k_{yw}^*\\ k_{wy}^* & k_w^*\end{bmatrix}_k$ the effective *k*-mode stiffness matrix

$\mathbf{1}^{\mathrm{T}} = \langle 1 \quad 0\rangle^{\mathrm{T}}$ the unit matrix, and (10)

$$k_{yk}^* = \Sigma k_{jk}^* = \Sigma(\omega_{jk}^2 M_{jk}^*) = M_{yk}^*\Sigma\overline{\omega}_{jk}^2$$

$$k_{wk}^* = \Sigma x_j^2 k_{jk}^* + \Sigma y_i^2 k_{ik}^* = M_{yk}^*\Sigma(x_j^2\overline{\omega}_{jk}^2 + y_i^2\overline{\omega}_{ik}^2)$$

$$k_{ywk}^* = k_{wyk}^* = \Sigma x_j k_{jk}^* = M_{yk}^*\Sigma x_j\overline{\omega}_{jk}^2$$

$$\overline{\omega}_{jk}^2 = \omega_{jk}^2\frac{M_{jk}^*}{M_{yk}^*}\ ,\ \overline{\omega}_{ik}^2 = \omega_{ik}^2\frac{M_{ik}^*}{M_{yk}^*}$$

The quantities $\overline{\omega}_{jk}$ and $\overline{\omega}_{ik}$ are now the 'effective' frequencies of the *j* and *i*-bents respectively and it is evident that when the lateral stiffness of a given building is composed by the same type of bents (e.g. flexural shear walls), they are respectively equal to ω_{jk} and ω_{ik}. Full description of the modal quantities derived by this analysis is given in Georgoussis (2009). It is worth noticing here that the matrix Eq. (9), for the first mode (k=1) single-story system, provides the response quantities of the first two modes of vibration of the real structure. Therefore, when the stiffness matrix of Eq. (9), for k=1, is decoupled, the first two modes of vibration (translational and rotational) are decoupled and in the case of a ground motion along the y-direction the response for a low height building will be practically translational. The stiffness matrix of Eq. (9) is decoupled when the term $k^*_{ywk}(=k^*_{wyk})$ is equal to zero. This condition specifies that the first mode center of rigidity (m_1-CR) of the corresponding single-story system coincides with CM. Generally, the x-coordination of m_1-CR (measured from CM) is given as

$$x_{m1-CR} = \frac{\Sigma(x_j \overline{\omega}_{j1}^2)}{\Sigma(\overline{\omega}_{j1}^2)} \tag{11}$$

And, evidently when $x_{m1\text{-}CR}$=0 there is a coincidence of m_1-CR and CM.

3. SYSTEMS ANALYZED

To illustrate the application and accuracy of the proposed method, the example structure shown in Fig. 4(a) was analyzed. This is an 8-story monosymmetric uniform building structure with a floor plan 15x10m, having two structural walls (Wa and Wb) and a moment resisting frame (FR) along the y-direction and a pair of wall bents (Wx) oriented in the axis of symmetry. The structural walls Wa and Wb are of cross sections 30x500cm and 30x400cm respectively, while the moment resisting frame FR consists of two 80x80cm columns, 6m apart, connected by beams of a cross section 35x70cm. The x-direction wall bents Wx are of the same dimensions as Wb and they are located symmetrically to CM at distances of 3m. The total mass per floor is m=120kNs^2/m, the radius of gyration about CM is r= 5.204m, the story height is 3.5m and the modulus of elasticity (E) is assumed equal to 25x10^6 kN/m^2. The center of mass at each floor lies on a vertical line passing through the centroid of the orthogonal floor plan at each level. To investigate the accuracy of the proposed method to a broader range of building structures, different structural configurations of the example structure are examined as follows: wall Wa and frame FR are assumed to be located at a fixed positions, the first on the left of CM in a distance equal to 4m and the second on the right of CM at a distance of 6m, while bent Wb is taking all the possible locations along the x-axis.

The accuracy of the proposed modified procedure to predict the resultant forces, in the case of a dynamic excitation (along the y-direction) characterized by a flat or the EC8-2004 recommended response spectrum (Fig. 4(b)) is examined by comparison with the results of a response spectrum analysis performed by means of the computer program SAP2000-V11. These results are also compared with those obtained by the methodology presented in earlier papers (Georgoussis 2009, 2012). In practical terms, the difference between the two approximate procedures is based on the grounds that at present the formulation of Eqs. (10) is based on the 'effective' frequencies $\overline{\omega}_{jk}$ and $\overline{\omega}_{ik}$, while in the older version this formulation is based on the real frequencies of the various bents ω_{jk} and ω_{ik} .

To apply the proposed method, the first pair of frequencies of the various bent-subsystems is required, and also their effective modal masses. Denoting with *M* the total mass of the structure (*M*=8*m*=960kNs^2/m), these quantities for wall Wa are as follows

ω_{wa1}=5.922/s, ω_{wa2}=34.278/s and $\overline{M}^*_{wa1} = M^*_{wa1}/M = 0.66$, $\overline{M}^*_{wa2} = 0.212$,

For wall Wb (and Wx): ω_{wb1}=4.261/s, ω_{wb2}=25.397/s and $\overline{M}^*_{wb1} = 0.658$, $\overline{M}^*_{wb2} = 0.208$.

For frame FR: ω_{f1}=3.529/s, ω_{f2}=11.771/s and $\overline{M}^*_{f1} = 0.774$, $\overline{M}^*_{f2} = 0.116$

The first two effective modal masses of the uncoupled structure, in the y-direction,, normalized in respect to the total mass, are respectively equal to $\overline{M}^*_{y1} = M^*_{y1}/M = 0.668$, $\overline{M}^*_{y2} = 0.202$. From these data, the effective frequencies given by the last of Eqs. (10), are equal to

For wall Wa: $\overline{\omega}_{wa1} = 5.886/s$, $\overline{\omega}_{wa2} = 35.116/s$, for wall Wb (and Wx): $\overline{\omega}_{wb1} = 4.229/s$, $\overline{\omega}_{wb2} = 25.771/s$ and for frame FR: $\overline{\omega}_{f1} = 3.789/s$, $\overline{\omega}_{f2} = 8.920/s$.

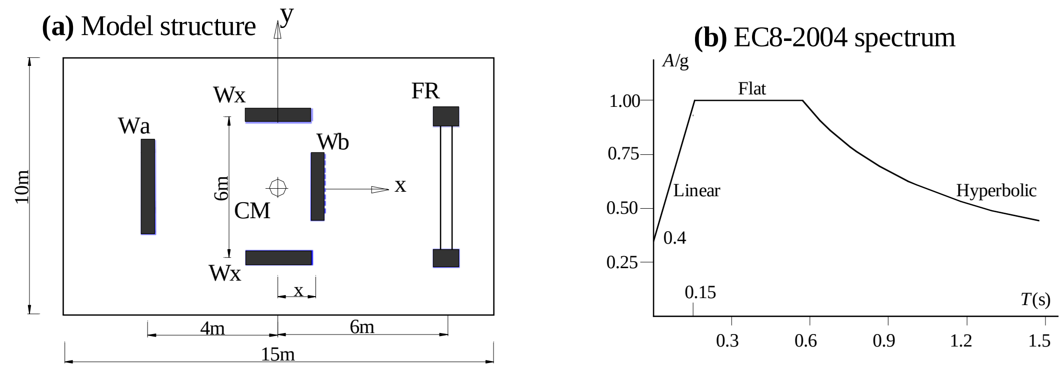

Fig. 4 (a) Floor plan of example structure; (b) Code-recommended design spectrum

4. DISCUSSION OF RESULTS

The first two periods of vibration of the example structure of Fig. 4(a), computed by the proposed modified method (T_m) for different locations of the frame system FR (indicated by the normalized coordinate $\overline{x} = x/r$), are shown in Fig. 5, together with the accurate computer values (T_{com}) and those obtained by the methodology presented by the first of the authors in earlier papers (T_{ea}). For the first mode of vibration, the approximate values T_{1m} (thick continuous green line) are exceeding, in all cases, the accurate computer values T_{1com} (thick continuous black line) but the error is less than 2.6%. A comparison between T_{1m} and T_{1ea} (thick continuous red line) shows that in all cases the modified method predicts more accurately the computer data (the T_{1m} line is closer to the T_{1com} curve than the T_{1ea} line). Similar are the data for the second mode of vibration (thick dotted lines in Fig. 5) although both the approximate results (T_{2m} and T_{2ea}) are closer to the accurate ones (T_{2com}). The maximum deviation of the T_{2m} curve is now 3%. In Fig. 5 are also shown the periods of the next two higher modes of vibration (third and forth) although these modes have a negligible effect on the overall response of low or medium height structures. For the third mode of vibration (thin continuous lines in Fig. 5) it is the procedure presented in the previous papers which presents higher accuracy: the T_{3ea} red curve is closer to the T_{3com} black curve than the green line of the T_{3m}values. In the fourth mode of vibration (thin dotted lines in Fig. 5) the proposed modified method is presenting a slightly higher accuracy, although the three curves (T_{4com}, T_{4m} and T_{4ea}) are almost coincident.

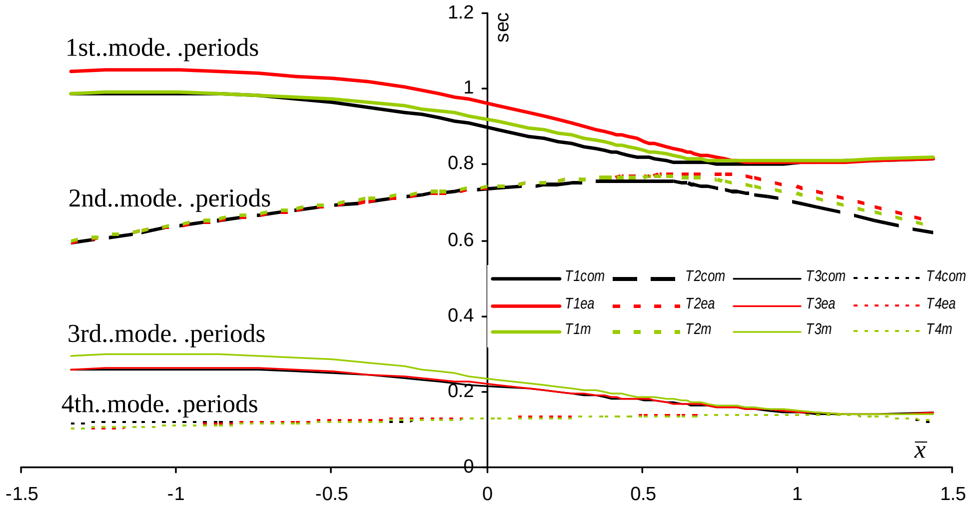

Fig. 5 Periods of vibration of the example structure

Base shears (in the y-direction) and torques, for the case of a flat spectrum, are shown in Figs. 6(a) and (b). Normalized shears and torques denoted as $\overline{V}_{m,1-2}$ and $\overline{T}_{m,1-2}$ (blue continuous lines) respectively, represent the approximate peak results obtained by the proposed modified procedure through the CQC rule on the grounds of the peak modal data derived from the analysis of the first mode (k=1) equivalent single-story system. The second mode (k=2) equivalent single-story system provides the corresponding data of the second pair of vibration modes, denoted as $\overline{V}_{m,3-4}$ and $\overline{T}_{m,3-4}$ (blue dashed lines) respectively. The total normalized resultant forces $\overline{V}_{m}$ and $\overline{T}_{m}$ (green lines) are computed from the sets $\overline{V}_{m,1-2}$, $\overline{V}_{m,3-4}$ and $\overline{T}_{m,1-2}$, $\overline{T}_{m,3-4}$ by means of the SRSS combination rule. All the aforesaid resultant forces are normalized in respect to the total shear, along the y-direction, of the uncoupled structure and base torques are also divided by r. In these figures are also shown (i) the data obtained by the methodology presented in earlier papers (total normalized resultant forces obtained as above and presented by the $\overline{V}_{ea}$ and $\overline{T}_{ea}$ red lines) and (ii) the accurate data $\overline{V}_{com}$ and $\overline{T}_{com}$ (black lines) given by the computer program SAP2000-V11 on the basis of the first 12 peak modal values combined according to the CQC rule (the damping ratio in each mode of vibration was taken equal to 5%). The results for the case of the Eurocode 8 response spectrum of Fig.4(b) are shown in Figs. 7(a) and (b).

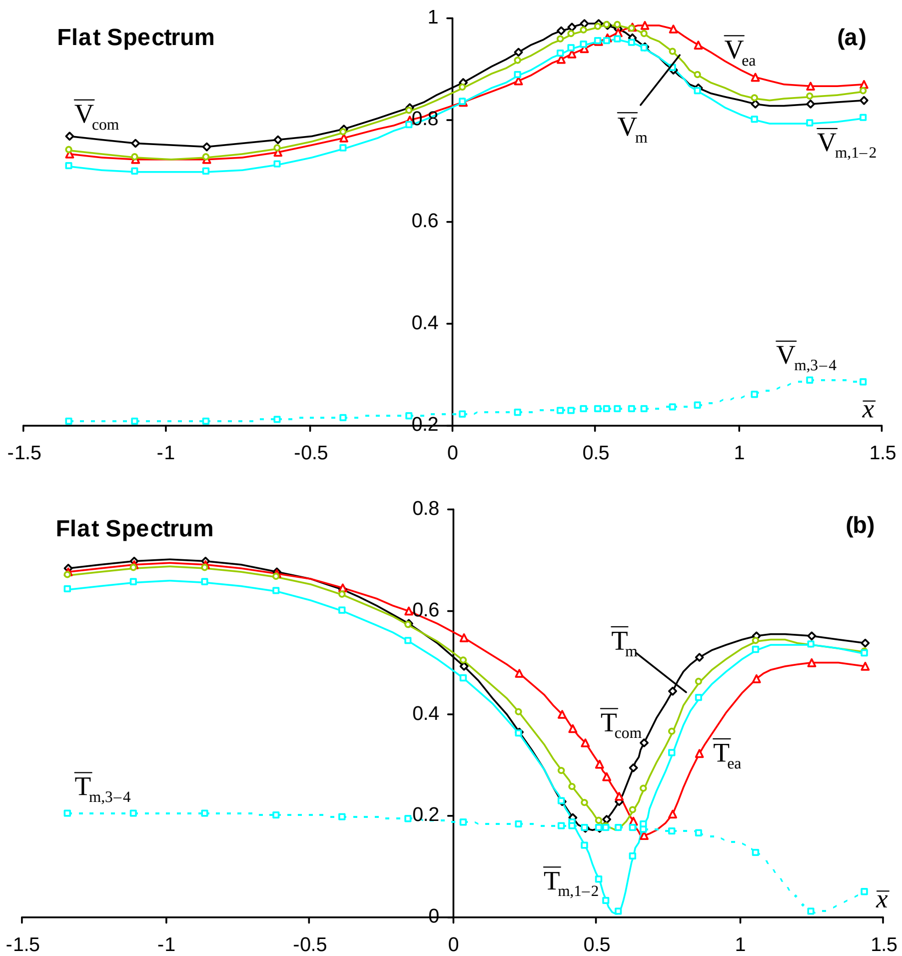

Fig. 6 Peak base shears: (a) and torques; (b) for the case of a flat spectrum

Envisaging Figs. (6) and (7), a first rough conclusion is that the proposed modified procedure provides data (green lines) closer to those of the computer analysis (black lines) than the approach presented by Georgoussis (2009, 2012) in earlier papers (red lines). The second overall observation is that the curves obtained from the Eurocode spectrum show an increased contribution of the second mode (*k*=2) equivalent single-story system (higher values of $\overline{V}_{m,3-4}$ and $\overline{T}_{m,3-4}$) on the total response of the example structures. This is attributed to two reasons: (i) the flat part of the response spectrum of

Figure 4(b) extents in the period range from 0.15 to 0.6sec, and (ii) the first two periods of vibration of all the examined structural configurations vary in the range 0.801- 0.989s for the first period, and in the range 0.593 – 0.756s for the second period of vibration.

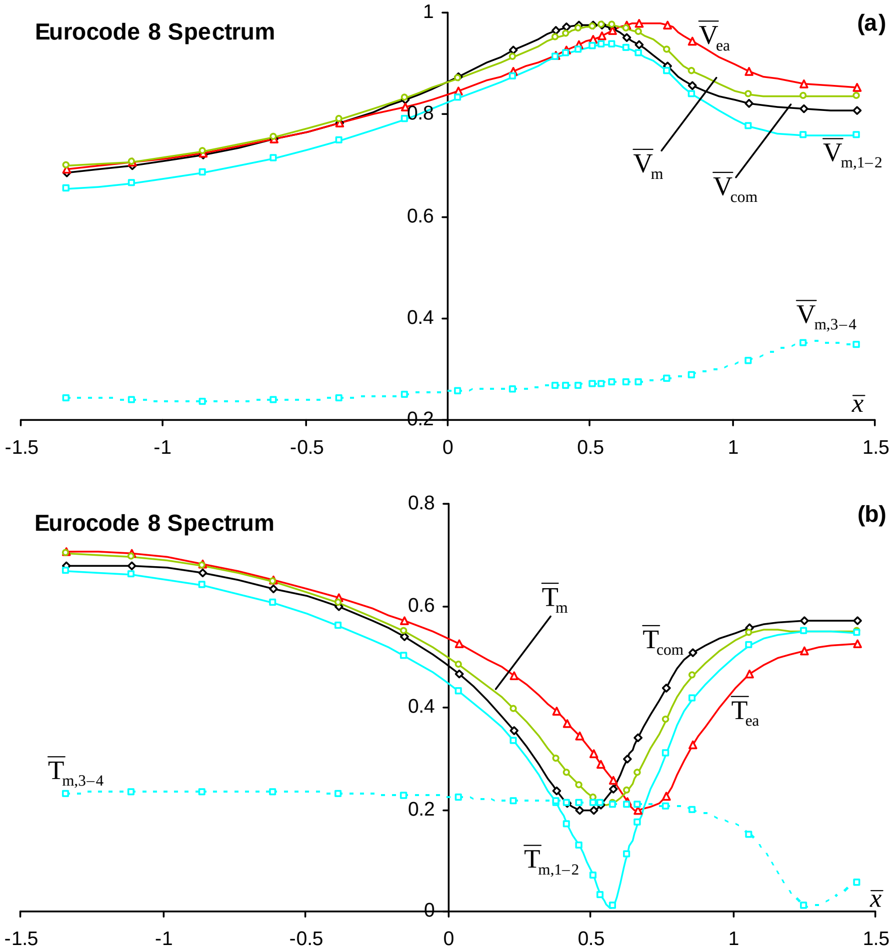

Fig. 7 Peak base shears (a) and torques (b) for the case of EC8-2004 spectrum

This means that in the case of the Eurocode spectrum the first mode peak acceleration is lower than that of the second mode and therefore the contribution of the latter mode is higher than in the case of the flat spectrum. It is reminded here that for buildings with the fundamental lateral vibration period in the acceleration sensitive region (that is, in the flat part of the response spectrum of Fig. 4(b)) the first two modes of vibration are adequate to obtain a reasonable estimate of the response of a building structure (Hejal and Chopra 1989b,c). It is of particular interest that the location of wall Wb which produces a normalized torque $\overline{T}_{m,1-2}$ equal to zero (which means that m_1-CR coincides with CM), represents the structural configuration which sustains minimum torsional response (minimum $\overline{T}_m$). For the assumed structural configurations, the aforementioned location of Wb is equal to $\bar{x} = 0.55$ and, as can be seen from Figs. 6(b) and 7(b), at this location of Wb the torsional response derived by the SAP2000 software is practically the minimum.

According to EC8-2004 (Clause 4.2.3.2), most of the structural configurations examined herewith should be considered as irregular buildings. There are basically two requirements which classify a building as a regular system: (i) that the distance between the centers of stiffness and mass is less than 30% of the 'torsional radius', and (ii) the 'torsional radius' is higher than the radius of gyration. According to paragraph (9) of the clause above, the centers of stiffness and the torsional radius may be calculated as those of the moments of inertia of the cross-sections of the vertical members. This means that none of the aforementioned requirements is satisfied when wall Wb 'moves' from the far left position (x=-7.5m) to the right of CM up to the location of x=3.6m. Besides, the second of the requirements above is not satisfied unless the wall Wb 'moves' to locations x>5.6m. Therefore, considering the closeness of the results derived by the proposed modified procedure with those of the computer SAP2000 program, it can be said that the proposed methodology may be applied with confidence to both regular and irregular in plan building systems. Regarding the definition of the center of stiffness of multi-story buildings and the long discussion about it in the relative literature, it must be noticed that the definition given by EC8-2004 is not a successful one. It would be expected that when this center (as it is described above) coincides with CM, the sustained torsion should be minimum. Using the data of the example structure, this means that minimum torsion would occur when the wall Wb is located at the far right side (x=7.5m) of the deck. The results presented in Figs. (6) and (7) simply indicate that minimum base torsion is attained when x=0.55x5.204=2.86m, that is, when the first mode center of rigidity (m1-CR) coincides with CM.

5. CONCLUSIONS

Frequencies and basic earthquake response (resultant base shears and torque) of eccentric, medium height uniform buildings, composed by dissimilar bents, can be estimated from the analysis of two equivalent, single-story, modal systems, the masses of which are determined from the first two vibration modes of the uncoupled multi-story structure and the stiffnesses of the resisting elements are determined from the corresponding individual bents when they are assumed to have, as planar frames, the mass of the complete structure. The proposed analysis improves the accuracy of the methodology developed in the past and provides the location of first mode center of rigidity with superior accuracy. The main property of this point is that when it lies close to the axis passing through the centers of floor masses, the response of a building structure is basically translational. Therefore, as it is quite easy to determine this point with simple hand calculations, the propose procedure can be used in the preliminary stage of a structural application to determine the optimum structural arrangement in terms of minimum torsional response.

To illustrate the application and accuracy of the proposed modified procedure on medium height mixed-bent-type buildings, which are classified by EC8-2004 as irregular buildings, a number of example structures are analyzed and these results are compared (i) with the accurate data received by the academic computer program

SAP2000-V11, and (ii) with the results of the procedure presented in previous papers (Georgoussis 2011, 2012). Acknowledging the limitations of this work and the needs for further studies on different building configurations, the following principal conclusions are drawn:

1. The proposed modified procedure provides with a reasonable accuracy the first pair of frequencies of common 8-story monosymmetric buildings with dissimilar bents, with an error less than 3%. Such a deviation is considered acceptable for design purposes and is less than that of the procedure presented in earlier papers.
2. Resultant base shears and torques are also accurately predicted from the analysis of the aforesaid equivalent, single-story, modal systems. In particular, the resultant base shear is basically obtained from the first mode equivalent system, but for the base torque the response of both systems should be taken into account.
3. The definition given by EC8-2004 about the center of stiffness of multi-story buildings is unsuccessful. It would be expected that when this point lies on the mass axis the torsional response of a building structure would be minimum. The results presented herewith show that this occurs only when the first mode center of rigidity lies in the mass axis.

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