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Identification of time-varying structures using TVAR and TVARV model based on B-spline wavelet

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ABSTRACT

An instantaneous frequency identification method for time-varying structures using TVAR(Time-Varying Auto Regressive) model based on B-spline wavelet is presented in the paper. The algorithm based on single or multiple sample points is deduced for structural instantaneous frequencies using acceleration response signals. The simulation studies are performed for a three degrees-of-freedom time-varying dynamic model. The structural instantaneous frequencies in the condition of periodically varying are identified by using single or multiple sample points. A confirmatory experiment is carried out using a cantilever beam with time-varying mass characteristics. The first three instantaneous frequencies are identified using the acceleration response signals. Results show that the proposed identification method is efficient and robust.
This paper focuses on the parameters identification for time varying structures. Based on B-spline wavelet on the interval，time-varying auto regressive model is established and the parameters identification procedure is deduced for structural instantaneous frequencies using accelerations responses signals. Depending on the different number of sample points, TVAR (time-Varying Auto Regressive) and TVARV (Time-Varying Auto Regressive for multi-Variable) are proposed. The simulation studies are performed based on a three degrees-of-freedom time-varying dynamic model. The structural instantaneous frequencies in the condition of periodically varying are identified based on the proposed two methods. At last, a confirmatory experiment is carried out using a cantilever beam with time-varying mass characteristics. The first three instantaneous frequencies are identified well using the acceleration response signals. And the results of the experiment demonstrate that the proposed two methods have very good validity, efficiency and practicality.
Keywords: Time-Varying System, System Identification, B-Spline Wavelet, TVAR Model, TVARV Model.

1. INTRODUCTION

Time sequence model was initially used for statistical analysis of the sunspot movements by Yule in 1927. But many processes are inherently time-varying and cannot effectively be characterized using time invariant models. One approach to characterize such processes is to employ the time-varying auto regressive moving average (TVARMA) model (Xu 2003) or the time-varying autoregressive (TVAR) model (Liu 2006).

Two main classes of methods can be used to solve the TVAR model estimation problem. The first uses recursive estimation of the time-varying coefficients, some of the most popular recursive algorithms are the least mean square (LMS) algorithm, the recursive least square (RLS) algorithm (Ljung 1990) and the Kalman filter algorithm (Morbidi 2008). And the second constrains the evolution of the coefficients to be linear or nonlinear combinations of some basis functions with appropriate properties. Generally, the associated time-varying coefficients are expanded as a finite sequence of predetermined basis functions (Chon 2005). Then the problem is reduced to time invariant coefficient estimation, where the unknown adjustable model parameters are those involved in the basis expansion. Hence, the time-varying modeling problem is simplified to basic functions selection and parameter estimation.

The choice of basic functions can significantly affect the performance of the parameter estimates. However, there is no guideline on how to choose the appropriate basis functions for a specific modeling problem. Therefore an attractive approach is to expand the time-varying coefficients using wavelets as the basis functions. B-spline wavelet on the interval has compact support and orthogonality characteristics. B-spline as piecewise polynomial functions were originally introduced as wavelet and scaling functions (Chui and Wang 1992). Furthermore B-spline wavelet was used in the system identification (Billings and Coca 1999).

Depending on the different number of sample points, TVAR and TVARV based on B-spline wavelet are proposed in the paper. The simulation study is performed based on a three degrees-of-freedom time-varying dynamic model using the structural acceleration response signals. The identification procedure has been shown to be effective in tracking time-varying parameters. And an experiment is carried out using a cantilever beam with time-varying mass characteristics. The first three instantaneous frequencies are identified well using the acceleration response signals. Results show that the proposed methods are valid, efficient and practicable.

2. TIME-VARYING AUTO-REGRESSIVE

Time-Varying Auto-Regressive (TVAR) can be represented as

$$x_t = -\sum_{i=1}^{p} \varphi_i(t) x_{t-1} + e_t \tag{1}$$

Where t is the time instant or sampling index of the signal x_t; the term e_t is the residual error accommodating the effects of measurement noise, and modeling noise that can be viewed as a stationary white noise sequence; $\{\varphi_i(t), i = 1,2,..., p\}$ is the time varying coefficient functions to be determined in the model; p is the order of AR model. $\varphi_i(t)$ can be expended by a set of orthogonal basis functions, such that the following expression

$$\varphi_i(t) = \sum_{j=0}^{m} \phi_{ij} g_j(t) \tag{2}$$

Where ϕ_{ij} represents the expansion parameters, and $\{g_j(t), j = 0,1,...,m\}$ is the set of basic functions. Define

$$A^T = [\phi_{10},...,\phi_{1m},...,\phi_{p0},...,\phi_{pm}] \tag{3}$$

$$X_{t-1}^T = [x_{t-1}g_0(t),\cdots,x_{t-1}g_m(t),\cdots,x_{t-p}g_0(t),...,x_{t-p}g_m(t)] \tag{4}$$

Such that model Eq. (1) can be rewritten as

$$x_t = -X_{t-1}^T A + e_t \tag{5}$$

2.1 Coefficient estimates

Based on Eq. (5), the following expression can be described using accelerations responses signals.

$$\begin{aligned} x_{p+1} + X_p^T A &= e_{p+1} \\ x_{p+2} + X_{p+1}^T A &= e_{p+2} \\ &\vdots \\ x_N + X_{N-1}^T A &= e_N \end{aligned} \tag{6}$$

The Eq. (6) can be written in a compact form

$$\Phi + \Theta A = E \tag{7}$$

where

$$\begin{aligned} \Phi &= [x_{p+1}, x_{p+2},\cdots, x_N]^T \\ \Theta^T &= [X_p, X_{p+1},\cdots, X_N] \\ E &= [e_{p+1}, e_{p+2},\cdots, e_N]^T \end{aligned} \tag{8}$$

In order to get the least squares estimate of matrix A, setting the residual sum of squares as the objective function.

$$J = E^T E = (\Phi + \Theta A)^T (\Phi + \Theta A) \tag{9}$$

Setting

$$\frac{\partial J}{\partial A} = 0 \tag{10}$$

Substituting Eq. (9) into Eq. (10), yields

$$\hat{A}_N = -(\Theta^T\Theta)^{-1}\Theta^T\Phi \tag{11}$$

In order to solve the estimated value of A quickly, recursive least square algorithm can be adopted. Setting

$$P_N = (\Theta^T\Theta)^{-1} \tag{12}$$

Such that the following expressions are obtained

$$\hat{A}_{N+1} = \hat{A}_N - P_N X_N \left(1 + X_N^T P_N X_N\right)^{-1}\left(x_{N+1} + X_N^T \hat{A}_N\right) \tag{13}$$

$$P_{N+1} = P_N - P_N X_N \left(1 + X_N^T P_N X_N\right)^{-1} X_N^T P_N \tag{14}$$

The new value is obtained constantly from the original value plus the revised one. Initial values P_0 and $\hat{A}_0$ need to be known, for example

$$\left.\begin{aligned} \hat{A}_0 &= 0 \\ P_0 &= \mu I \end{aligned}\right\} \tag{15}$$

Where $\mu >> 1$ and $\mu = 10^4$. I is an unit matrix whose dimension is $[(m+1)\cdot p]\times[(m+1)\cdot p]$.

2.2 parameter identification

The natural frequency and damping ratio of the system can be calculated after obtaining the coefficients of the AR model. Set the coefficient of the time-varying AR model to be constant in the sampling time interval Δt, that is $t_d = d\Delta t$ (d=1, 2… N，N is total sampling points). Therefore the following homogeneous differential equation is

$$1 + \varphi_1(t_d)s^{-1} + \varphi_2(t_d)s^{-2} + \cdots + \varphi_p(t_d)s^{-p} = \sum_{i=0}^{p}\varphi_i(t_d)s^{-i} = 0 \tag{16}$$

$s_i(t_d)$ solved in the Eq. (16) is the pole of the system transfer function, which contains the information of natural frequency and damping ratio.

$$\begin{cases} s_i(t_d) = e^{(-\xi_j(t_d)\omega_{nj}(t_d) + j\sqrt{1-\xi_i^2(t_d)})\Delta t} \\ s_i(t_d)^* = e^{(-\xi_j(t_d)\omega_{nj}(t_d) - j\sqrt{1-\xi_i^2(t_d)})\Delta t} \end{cases} \tag{17}$$

Therefore natural frequency $f_i(t_d)$ and damping ratio $\xi_i(t_d)$ of the system are calculated as follow respectively.

$$f_i(t_d) = \frac{1}{2\pi}\frac{\left|\ln(s_i(t_d))\right|}{\Delta t} \tag{18}$$

$$\xi_i(t_d) = \sqrt{\frac{1}{1+(\frac{\mathrm{Im}(\ln s_i(t_d))}{\mathrm{Re}(\ln s_i(t_d))})^2}} \tag{19}$$

2.3 Time-Varying Auto-Regressive for multi-Variable

The time-varying auto-regressive for multi-variable (TVARV) model can be denoted as the following equation.

$$X_t = -\sum_{i=1}^{p}\Phi_i(t)X_{t-i} + E_t \quad (t=1，2,\ldots,\mathrm{N}) \tag{20}$$

Where $X_t = [x_{1t}, x_{2t}, \ldots, x_{lt}]^T$ and $E_t = [e_{1t}, e_{2t}, \ldots, e_{lt}]^T$ are the sampled measured output and error signals. $\Phi_i(t)$ (i=1, 2… *p*) are the time varying parameters to be determined whose dimension is $l \times l$; *p* is the model order.
The $\Phi_i(t)$ can be expended by a set of orthogonal basis functions, such that the following expression

$$\Phi_i(t) = \sum_{j=0}^{m}\varphi_{ij}g_j(t) \tag{21}$$

Where $g_j(t)$ (j=0, 1… *m*) are basis functions and φ_{ij} is the coefficient matrix which can be written as

$$\varphi_{ij} = \begin{bmatrix} \varphi_{11j} & \varphi_{12j} & \cdots & \varphi_{ilj} \\ \varphi_{21j} & \varphi_{22j} & \cdots & \varphi_{2lj} \\ \cdots & \cdots & \cdots & \cdots \\ \varphi_{l1j} & \varphi_{l2j} & \cdots & \varphi_{llj} \end{bmatrix} \tag{22}$$

For obtaining $\Phi_i(t)$, setting

$$Q^T = [\varphi_{10}, \cdots, \varphi_{1m}, \cdots, \varphi_{p0} \cdots \varphi_{pm}]_{l\times[l(m+1)\cdot p]} \tag{23}$$

$$Y_{t-1}^T = [X_{t-1}^T g_0(t), \cdots, X_{t-1}^T g_m(t), \cdots, X_{t-p}^T g_0(t), \cdots, X_{t-p}^T g_m(t)]_{1\times[l(m+1)\cdot p]} \tag{24}$$

Therefore model Eq. (20) can be rewritten as

$$X_t^T = -Y_{t-1}^T Q + E_t^T \tag{25}$$

After solving Eq. (25) to obtain the coefficients of the AR model, the following homogeneous differential equation can be described as

$$\det(I+\Phi_1(t_d)Z^{-1}+\Phi_2(t_d)Z^{-2}+\cdots+\Phi_p(t_d)Z^{-p})=0 \tag{26}$$

Matrix $G(t_d)$ that has the same eigenvalues with Eq. (26) is composed to avoid solving nonlinear equations.

$$G(t_d)=\begin{bmatrix} 0 & I & 0 & 0 & \cdots & 0 \\ 0 & & I & 0 & \cdots & 0 \\ \cdots & & & & \cdots & \\ -\Phi_p(t_d) & -\Phi_{p-1}(t_d) & \cdots & \cdots & \cdots & -\Phi_1(t_d) \end{bmatrix} \tag{27}$$

Therefore the eigenvalues can be calculated by the eigenvalue decomposition of $G(t_d)$. And the natural frequencies and damping ratios of the system are calculated using Eq. (17), Eq. (18) and Eq. (19).

3. B-SPLINE WAVELETS

The TVAR model includes time dependent auto regressive coefficients which can be expanded by a set of orthogonal basis functions. Thus the time-varying problem is transformed into a time-invariant one. Conventionally, the basic functions have been chosen to be polynomials including Chebyshev and Legendre types, prolate spheroidal sequences which are the best approximation to band limited functions (Zou and Chon 2004).

In the paper, B-spline wavelet on the interval (BSWI) is chosen as the basis functions. B-spine scaling functions $\phi_{m,k}^{j}(\xi)$ (m is the order and j is scale) can be expressed (Guan 1995) as follows.

$$\phi_{m,k}^{j}(\xi)=\begin{cases}\phi_{m,k}^{l}\left(2^{j-l}\xi\right) & k=-m+1,\cdots,-1; \quad 0 \quad boundary \\ \phi_{m,2^j-m-k}^{l}\left(2^{j-l}(1-\xi)\right) & k=2^j-m+1,\cdots,2^j-1; \quad 1 \quad boundary \\ \phi_{m,0}^{l}\left(2^{j-l}\xi-2^{-l}k\right) & k=0,\cdots,2^j-m; \quad Internal\end{cases} \tag{28}$$

Choose $\phi_{4,k}^{4}(\xi)$ as scaling function which has 19 expressions in total, *i.e.*, $\phi_{4,-3}^{4}(\xi)$、$\phi_{4,-2}^{4}(\xi)$、$\phi_{4,-1}^{4}(\xi)$ are on 0 boundary , $\phi_{4,13}^{4}(\xi)$、$\phi_{4,14}^{4}(\xi)$、$\phi_{4,15}^{4}(\xi)$ are on 1 boundary and $\phi_{4,0}^{4}(\xi)$、$\phi_{4,1}^{4}(\xi)$、$\phi_{4,2}^{4}(\xi)$、$\phi_{4,3}^{4}(\xi)$、$\phi_{4,4}^{4}(\xi)$、$\phi_{4,5}^{4}(\xi)$、$\phi_{4,6}^{4}(\xi)$、$\phi_{4,7}^{4}(\xi)$、$\phi_{4,0}^{4}(\xi)$、$\phi_{4,9}^{4}(\xi)$、$\phi_{4,10}^{4}(\xi)$、$\phi_{4,11}^{4}(\xi)$、$\phi_{4,12}^{4}(\xi)$are internal wavelets. These scaling functions are shown in Fig. 1.

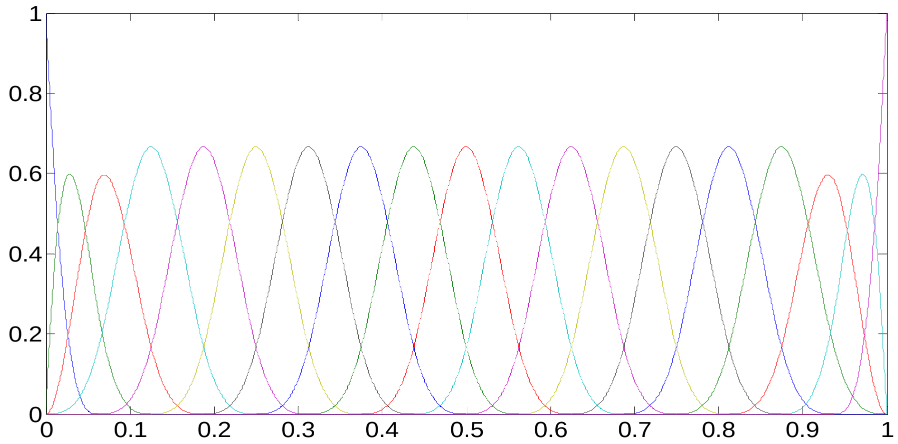

Fig.1 the scaling function of B-spline

4. SIMULATION EXAMPLE

A system of three degrees-of-freedom is shown in Fig. 2.

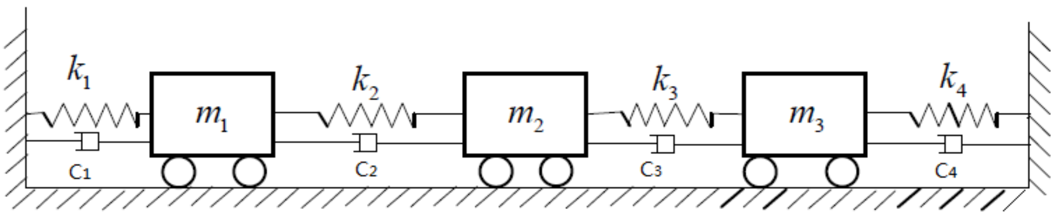

Fig.2 three degrees-of-freedom dynamic model

Where $m_1 = m_2 = m_3 = 1kg$, $c_1 = c_2 = c_3 = c_4 = 0.5Ns/m$ and $k_1 = k_3 = k_4 = 4000N/m$.
For comparison of the results of TVAR a TVARV, the instantaneous frequencies are identified in the condition of periodically varying. Sampling frequency *f* = 200Hz and total time *T* = 30s. The result of TVAR is obtained only depend on single acceleration response signal, while TVARV using all three degree-of-freedom acceleration signals. Fig. 3 - Fig. 5 show the structural instantaneous frequencies identified by the proposed two methods.

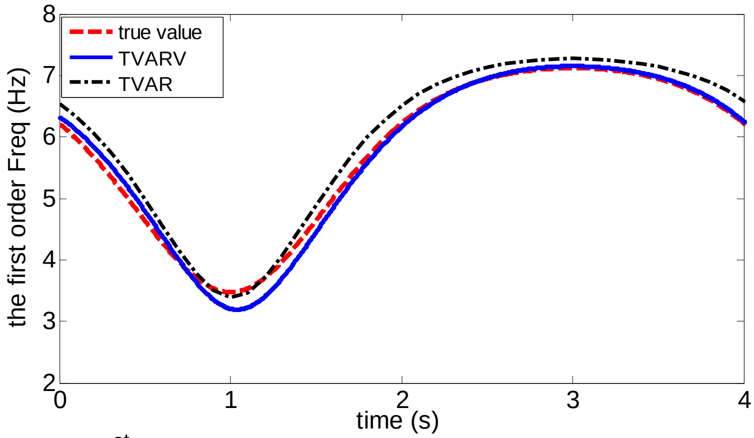

Fig.3 Comparison of 1st instantaneous frequency identified using TVAR and TVARV

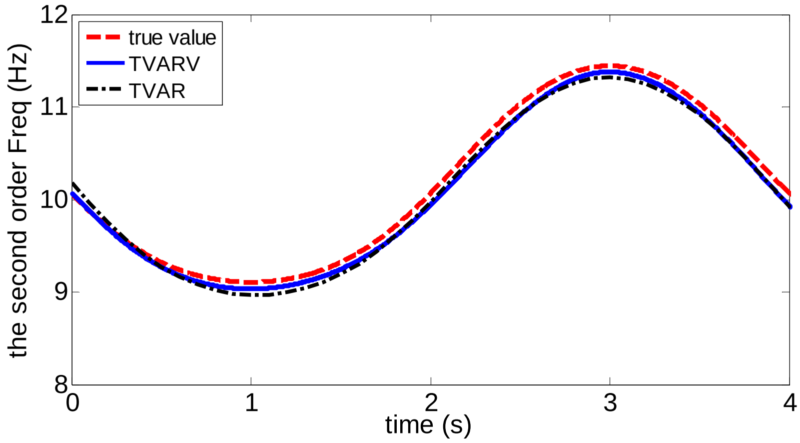

Fig.4 Comparison of 2nd instantaneous frequency identified using TVAR and TVARV

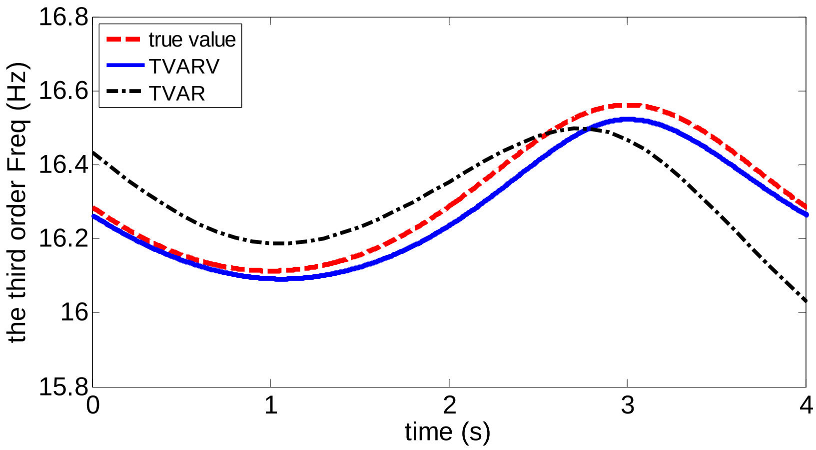

Fig.5 Comparison of 3rd instantaneous frequency identified using TVAR and TVARV

In order to compare the results of TVAR and TVARV further, define the Mean Absolute Percentage Error (MAPE) as follow

$$MAPE = \frac{1}{N_L}\sum_{i=1}^{N_L}\left|\frac{f_i - \tilde{f}_i}{f_i}\right| \times 100\% \quad (29)$$

Where N_L represents the total number of sample points, f_i is the true value of instantaneous frequency and $\tilde{f}_i$ is identified value. The MAPE are shown in Table 1.

Table 1 Comparison of identified error between TVAR and TVARV

	MAPE（%）		
Order	1	2	3
TVAR	1.8156	1.0346	2.0935
TVARV	1.5745	0.9751	1.3636

The simulation results demonstrate that the proposed two methods can follow the true parameter variation very well. And TVARV gives better results than TVAR which can be explain as follow: TVARV uses acceleration response signals in all degrees of freedom that contain more modal information.

Fig.6 A time-varying cantilever beam test

5. EXPERIMENTAL STUDY

5.1 Experimental Design

An aluminum cantilever beam which is fixed on the right end is shown in Fig.6. Its dimension is 1300×80×20 mm, the Young's modulus is $\mathrm{E} = 7\times10^{10}\ \mathrm{N/m^2}$ and density is $\rho = 2700\ \mathrm{kg/m^3}$. The left end is bonded an iron square box with 7 powerful magnets stuck to its bottom and surrounding. The box can be called as a ‘magnetic box’. In test process, 1kg of iron flour is slowly poured into the ‘magnetic box’ with a uniform speed through a funnel. Because of the effect of magnetic field, the iron flour will be firmly stuck together with the ‘magnetic box’. Therefore, the cantilever beam can be considered as a time-varying dynamic system.

5.2 Experiment Process

The 1kg of iron flour is divided into ten bags averagely which is poured into the ‘magnetic box’ in turn. The beam is divided into 13 units. Three acceleration response signals of No.5, 6 and 8 points are sampled by the data acquisition card of NI 4431 as shown in Fig.7.

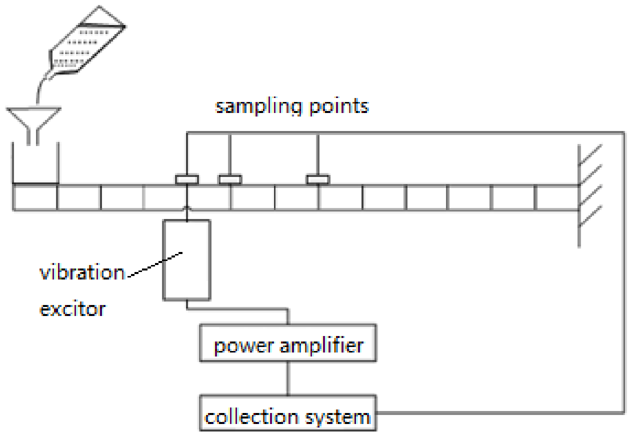

Fig.7 The sketch chart of cantilever beam vibration test

If the beam with some bags of iron flour and no change is called as a ‘frozen state’, The cantilever beam has 11 ‘frozen states’ totally together with no iron flour state (the ‘magnetic box’ is empty). Each ‘frozen state’ is a time-invariant system. 33 FRF of 3 measuring points can be obtained by the data acquisition card, which can be regarded as the reference for time-varying system.

5.3 Experimental Result

For the time-varying system, the sampling frequency is 1024Hz and the sampling time is 30s. The first three instantaneous frequencies are identified are shown in Fig.8 - Fig.10. TVAR is based on the acceleration response signals of No.5 sampling point while TVARV uses signals of three sampling points.

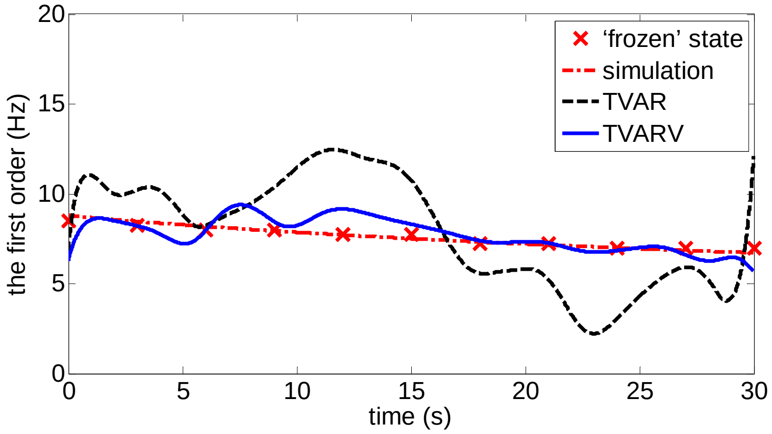

Fig.8 The first identified time-varying frequencies

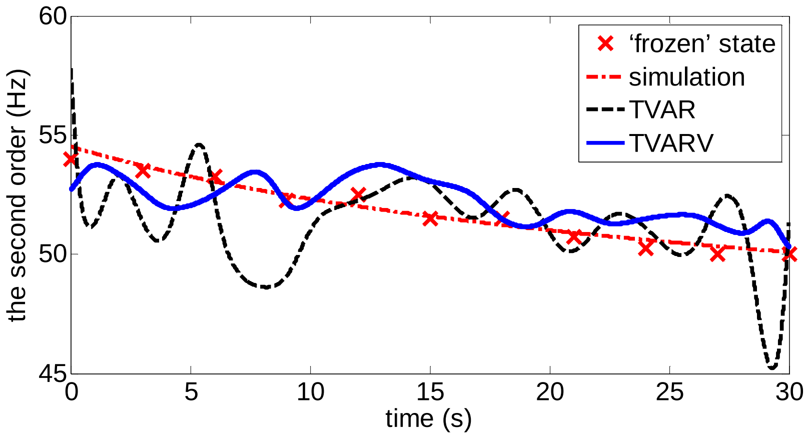

Fig.9 The second identified time-varying frequencies

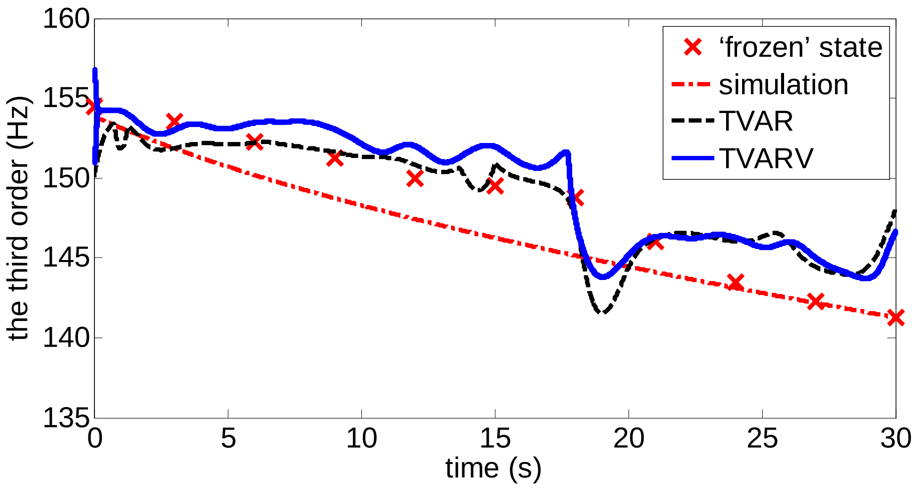

Fig.10 The third identified time-varying frequencies

In the Fig.8 - Fig.10, the identified time-varying instantaneous frequencies match up the reference values well. And TVARV gives better results than TVAR.

6. CONCLUSION

An instantaneous frequency identification method for time-varying structures based on B-spline wavelet is presented in the paper. Depending on single or multiple of acceleration sampling signals, Structural instantaneous frequencies can be identified quickly using TVAR or TVARV algorithm respectively. The simulation study and experimental results show that the proposed methods are effective in tracking time-varying frequencies.

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REFERENCES

1. Xu, X. Z., Zhang, Z. Y. and Hua, H. X. (2003), "Identification of Time-Variant Modal Parameters by a Time-Varying Parametric Approach," *Acta Aeronautica et Astronautica Sinica*, **24**(5), 230-233.
2. Liu, L. L., Liu, H. Z. and Wu, Z. Y. (2006), "Modal Parameter Identification of Time Varying Vibration System Using the Time Varying Multivariate Prony Method," *Chinese Journal of Mechanical Engineering*, **42**(1), 134-138.
3. Ljung, L. and Gunnarsson, S. (1990), "Adaptation and tracking in system identification — A survey," *Automatica*, **1**(26), 7–21.
4. Morbidi F., Garulli A., Prattichizzo D., Rizzo C. and Rossi S. (2008), "Application of Kalman filter to remove TMS-induced artifacts from EEG recordings," *IEEE Trans. Control Syst. Technol*., **16**(6), 1360–1366.
5. Chon, K. H., Zhao, H., Zou, R. and Ju, K. (2005), "Multiple time-varying dynamic analysis using multiple sets of basic functions," *IEEE Trans. Biomed. Eng.*, **52**(5), 956–960.
6. Chui, C.K. and Wang, J.H. (1992), "On compactly supported spline wavelets and a duality principle," *transactions of American Mathematical Society*, **330**, 903-915.
7. Billings, S.A. and Coca, D. (1999), "Discrete wavelet models for identification and qualitative analysis of chaotic systems," *International Journal of Bifurcation and Chaos*, **9**(7), 1263-1284.
8. Zou, R. and Chon, K. H. (2004), "Robust algorithm for estimation of time-varying transfer functions," *IEEE Trans Biomed Eng*, **51**(2), 219–228.
9. Guan, L. T. (1995), "Wavelet Interpolation and Decomposition in a Finite Interval with Boundary Conditions," *Journal of Engineering Mathematics*, **12**(3), 1-9.