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Effect of plastic anisotropy on the limit load of highly under-matched joints: A conceptual approach to plane strain problems

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ABSTRACT

The present paper is concerned with a general approach to determine semi-analytical limit loads for a class of highly under-matched plastically anisotropic welded joints. The definition for highly under-matched welded joints is that the weld is much softer than the base material. Therefore, plastic deformation is confined within the weld whereas the base material is rigid. The approach proposed is based on two principles. First, the general kinematically admissible velocity proposed accounts for singular behavior of real velocity fields near bi-material interfaces. Second, the thickness of the weld is usually much smaller than other geometric dimensions of specimens. Therefore, a linear through-thickness distribution on the velocity component normal to the weld is assumed. ……

1. INTRODUCTION

The limit load is an essential input parameter in many flaw assessment procedures (Zerbst et al., 2000). A comprehensive overview of limit load solutions for structures with defects available at the time of writing can be found in Miller (1988). An overview of limit load solutions for highly under-matched welded joints including joints containing cracks has been given in Alexandrov (2012). A distinguished feature of this class of welded joints is that the weld is much softer than the base material. In particular, plastic deformation is solely confined within the weld whereas the base material is rigid. Such structures are of practical interest (Hao et al., 1997). The present paper concerns with a general method to build up kinematically admissible velocity fields for a class of highly under-matched plastically anisotropic welded joints. It is worthy of note that elastic properties have no effect on the limit load (Drucker et al., 1952). The approach proposed is based on two principles. First, it is known that the real velocity field is singular in the vicinity of envelopes of characteristics (Alexandrov and Richmond, 2001, Alexandrov and Jeng, in press). In particular, the equivalent strain rate involved in the formulation of the upper bound theorem of plasticity (Hill, 1950) follows an inverse square root rule near such surfaces and, therefore, approaches infinity. In the case of highly under-matched welded joints envelopes of characteristics coincide with the bi-material interface. It is advantageous to account for the singular behaviour of the real velocity fields in kinematically admissible velocity fields. Second, the thickness of the

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weld is usually much smaller than other geometric dimensions of specimens. It is therefore natural to assume a linear through-thickness distribution on the velocity component normal to the weld. The first principle has been adopted in Alexandrov and Richmond (2000) to propose a general method to evaluate the tensile strength of adhesive plastic layers of arbitrary simply connected contour. The method has been applied to estimate the effect of three-dimensional deformation on the limit load for a highly under-matched welded joint of rectangular cross-section in Alexandrov (1999). The second principle has been ignored in Alexandrov and Richmond (2000). On the other hand, the exact analytic solution for compression of a thin rigid/plastic layer between two rough parallel plates predicts a linear through-thickness distribution of the velocity component normal to the layer (Hill, 1950). A similar distribution of this velocity component appears in compression of an anisotropic layer (Collins and Meguid, 1977). It is therefore reasonable to account for such solution behavior in kinematically admissible velocity fields under consideration.

2. CONCEPTUAL APPROACH

The class of structures under consideration is restricted to highly under-matched welded joints under plane strain conditions. The definition for highly under-matched welded joints assumes that plastic deformation at the instant of plastic collapse is localized within the weld whereas the base material is rigid. A consequence of such a flow pattern is that the yield stress of the base material has no effect of the limit load. Therefore, the mis-match factor, which is considered to be an important parameter of welded joints, is not involved in the present formulation. Numerous limit load solutions for highly under-matched joints show that the velocity vector is discontinuous across a significant portion of the bi-material interface (Alexandrov, 2012). In most cases, these velocity discontinuity surfaces are envelopes of characteristics. It is known (Alexandrov and Jeng, in press) that the real velocity field is singular in the vicinity of envelopes of characteristics in plane strain flow of anisotropic materials. The present study is restricted to plane strain problems for orthotropic materials obeying the quadratic yield criterion proposed in Hill (1950). It is advantageous to take into account the singular behavior of the real velocity field found in Alexandrov and Jeng (in press) in kinematically admissible velocity fields. Introduce a local Cartesian coordinate system (α, β, z) whose z – axis is orthogonal to the plane of flow and β - axis is orthogonal to the bi-material interface. The non-zero strain rate components in this coordinate system are denoted by $\xi_{\alpha\alpha}$, $\xi_{\beta\beta}$ and $\xi_{\alpha\beta}$. It has been shown in Alexandrov and Jeng (in press) that the shear strain rate in this coordinate system follows an inverse square root rule in the form

$$\left|\xi_{\alpha\beta}\right| = \frac{E}{\sqrt{\beta}} + o\left(\frac{1}{\sqrt{\beta}}\right) \tag{1}$$

as $\beta \to 0$. It is evident that β is the normal distance to the bi-material interface. In what follows this distance will be denoted by s. In Eq. (1), E is independent of β. In the case of the quadratic yield criterion proposed in Hill (1950), the plastic work rate is given by

$$W = 2T\sqrt{\left(1-c\right)\xi_{\alpha\alpha}^2 + \xi_{\alpha\beta}^2} \tag{2}$$

It has been taken into account here that $\xi_{\alpha\alpha} = -\xi_{\beta\beta}$ due to plastic incompressibility. In Eq. (2)

$$\begin{gathered} c = 1 - \frac{F+G}{4T^2\left(FG+GH+HF\right)}, \\ 2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}, \quad 2G = \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}, \quad 2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}. \end{gathered} \tag{3}$$

Here X, Y, and Z are the tensile yield stresses in the α - , β - and thickness directions, respectively. Also, T is the shear yield stress in the $\alpha\beta$ -plane. The upper bound theorem reads

$$\iint_{S_v} \left(t_i v_i\right) dS \le \iiint_{\Omega} W d\Omega + \iint_{S_d} \tau_s \left\|\left[u_\tau\right]\right\| dS \tag{4}$$

where Ω is the volume of material loaded by prescribed velocities v_i over a part S_v of its surface, $\left\|\left[u_\tau\right]\right\|$ is the amount of velocity jump across the velocity discontinuity surface S_d and τ_s is the magnitude of the shear stress component referred to the slip-lines. It is known that (Hill, 1950)

$$\tau_s = T\sqrt{1 - c\sin^2 2\phi} \tag{5}$$

where   is the anti-clockwise orientation of the slip-line to the α - axis. The plastic work rate and $\left\|\left[u_\tau\right]\right\|$ involved in Eq. (4) should be found using any kinematically admissible velocity field u_i . Since the normal velocity must be continuous across any velocity discontinuity surface, the velocity component u_τ is tangent to this surface. Equation (4) enables the stresses t_i applied over S_v to be evaluated. If the only unknown load is a tensile force *F* then Eq. (4) can be transformed to

$$F_u V = \iiint_{\Omega} W d\Omega + \iint_{S_d} \tau_s \left\|\left[u_\tau\right]\right\| dS \tag{6}$$

where F_u is the upper bound on the magnitude of *F* at plastic collapse and *V* is the velocity of the point at which the force is applied. It has been assumed here that the vectors **F** and **V** are collinear. It follows from Eqs. (1) and (2) that the volume integrals in Eqs (4) and (6) are improper. However, it is easy to show convergence.

The weld is idealised by a narrow layer of constant thickness. A typical weld configuration is shown in Fig. 1. The thickness of the weld is 2*H* and its width is 2*L*. There are two axes of symmetry coinciding with the axes of Cartesian coordinates (*x*, *y*). For a sake of simplicity, it is assumed that the boundary value problem is symmetric relative to these axes. It is therefore sufficient to get the solution in the domain $x \geq 0$ and $y \geq 0$. There should a rigid zone in the vicinity of the *x*-axis. This zone sticks to the base material whose motion is prescribed. Therefore, the motion of the rigid zone is prescribed as well. There are two velocity discontinuity curves, 0*b* and *bc* (Fig. 2). The shape of the velocity discontinuity curve 0*b* should be found from the solution.

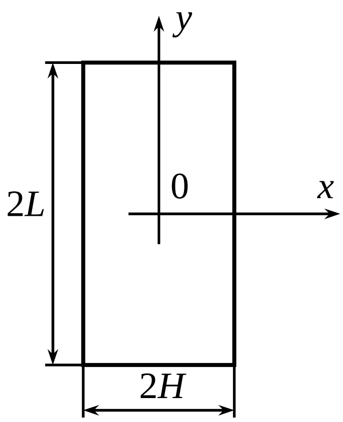

Fig. 1 Idealised weld configuration

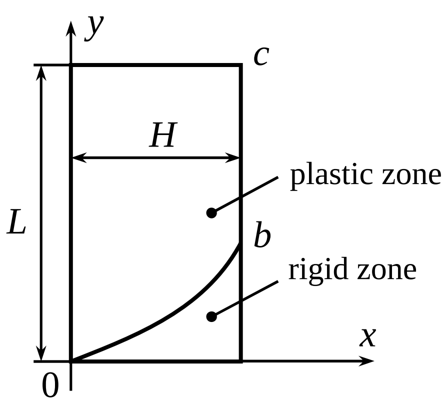

Fig. 2 Flow pattern

Let u_x and u_y be the velocity components in the Cartesian coordinate system. Because of symmetry, one of the velocity boundary conditions is

$$u_x = 0 \tag{7}$$

at $x = 0$. In the case under consideration $s = H - x$. Therefore, it follows from Eq. (1) that

$$u_y = U_0 + \frac{U_1}{\sqrt{H-x}} + o\left(\frac{1}{\sqrt{H-x}}\right), \quad x \to H \tag{8}$$

where U_0 and U_1 may depend on *y*. By assumption, u_x is a linear function of *x*. Taking into account Eq. (7) it is possible to get

$$u_x = \lambda\left(\frac{y}{H}\right)\frac{x}{H}. \tag{9}$$

where $\lambda\left(y/H\right)$ is an arbitrary function of its argument. Using Eqs. (8) and (9) it is possible to find the shape of 0b and the magnitude of $\left[u_\tau\right]$ across the velocity discontinuity curves. Then, Eq. (5) can be used to determine τ_s. Substituting this value of τ_s along with Eqs.(8) and (9) into Eq. (6) gives an upper bound on F. The parameters involved in Eq. (8) should be found by minimizing the right hand side of Eq. (6).

3. CONCLUSIONS

A general method to build up kinematically admissible velocity fields satisfying Eq. (1) has been proposed. The method is a generalization of the method for isotropic materials (Alexandrov, 2012). Since the latter has been successfully used for a great number of configurations, it is expected that the new method will result in accurate limit load solutions for anisotropic materials. …

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