The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Brittle fracture on crack initiation beneath bearing surface of layered substrate

Y. J. Xie[1)] and X. H. Wang[2)]

[1), 2)] *Department of Mechanical Engineering, Liaoning Shihua University, Fushun, LN, 113001, P. R. China*
[1)] *yjxie@lnpu.edu.cn*

ABSTRACT

When a rigid, flat-ended uniform indenter is pressed on the surface of a layered substrate, a singular stress field and K-dominant region should arise adjacent to the indentation edges. The indentation stress intensity factor (ISIF) is also a fracture parameter similar to the mode I crack, which represents the intensification of the stress fields. In present article, a method to formulize the ISIF for the singular stress fields induced by indentation is proposed based on the conservation integral. The energy release rate of crack initiation from the indentation edges is derived. The K-based critical condition for crack initiation is found also.

1. INTRODUCTION

It has been well known that if a rigid spherical or cylindrical body is pressed into a large flat glass block, a cone crack will form suddenly at a characteristic load (Frank and Lawn 1967). This kind of boundary cracking has been a rich and productive topic of research over the past several decades, and remains so because of its relevance to contact failures of materials such as bi- and tri-layer structures with brittle coatings in many recent bio-materials applications (Ford et al. 2004). It has been proven to be difficult to establish a satisfactory analytical solution to quantitatively describe the crack initiation at the indenter edge. Generally speaking, the most critical regions for possible unstable growth of a flaw are always where there is a localisation of stress, with steep stress gradients, and local tension. The failure beneath the bearing surface under indentation by a rigid and flat-ended indenter is another typical example of boundary cracking induced by the singular stress fields except mode I crack. Actually, knowing the material's fracture toughness, it is possible to predict the critical load at which fracture occurs.

The present work will focus on the surface critical brittle cracking of the layered substrate under indentation. Actually, singular stress fields appear at not only the crack tip but also the edge of the indentation. The damage caused by such stress fields refers to the crack extension for the crack configuration and the crack embryo for the indentation. The SIF for crack and ISIF for indentation representing the stress intensification are fracture parameter for both cases, which is the main causation to lead the engineering components to failure. For crack problem, various methods and theory have been developed to determine the SIF and the critical cracking condition. However, the literatures on the calculation of the ISIF and the critical cracking condition for indentation were rarely reported. As we all know, the ISIF for indentation is as important as cracks. Indentation is different from the crack after all. Some behaviours of fracture for the indentation are other than the cracks (Xie and Hills 2007). The further investigations on the failure analysis are constructive for the contact problems, including the indentation on the layered substrates which can be used to analyses the micro-behaviour of the coating materials.

2. CONTACT PROBLEM

The indentation geometry considered is illustrated in Fig.1(a) and Fig.2. A rigid flat-ended indenter of half-width *l* is pressed by a normal load, *P*, onto the surface of a frictionless half-plane, having a Poisson's ratio μ . The contact problem may readily be solved in closed form, and gives rise to the following contact pressure distribution (Nadai 1963)

$$p(x_1) = -\frac{P}{\pi\sqrt{l^2 - x_1^2}} \tag{1}$$

Make the change of co-ordinates $x_1 = r - l$, and expand this solution, for small *r*, using the binomial distribution. This shows that the pressure in the neighborhood of the indenter corner varies as

$$p(x_1) = \frac{P}{\pi}\left(\frac{1}{\sqrt{2lr}} + \cdots\right) \tag{2}$$

whilst the requirement that the contact be frictionless means that the shear traction is zero everywhere along the contact boundary. It will be noted that these stress components are precisely the same as those arising along the line $x_2 = 0$ for a crack suffering Mode I loading shown in Fig.1(b). In fact, because the boundary conditions along the line $x_2 = 0$ are identical in the contact and crack problem, the internal state of stress everywhere within the two problems is the same, which can be defined as the Mode I indentation. The expressions of the stress next to the corners of the indenter can be given by a classical crack-tip field solution (Nadai 1963), i.e.,

$$\begin{pmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{r\theta} \end{pmatrix} = -\frac{K_{I-ind}}{\sqrt{2\pi r}} \begin{pmatrix} \cos\frac{\theta}{2}\left(1 + \sin^2\frac{\theta}{2}\right) \\ \cos^3\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \end{pmatrix} \tag{3}$$

where the indentation stress intensity factor for the crack problem is defined in the usual way, and for the half-plane contact problem it is given by

$$K_{I-ind} = \frac{P}{\sqrt{\pi l}} \,. \tag{4}$$

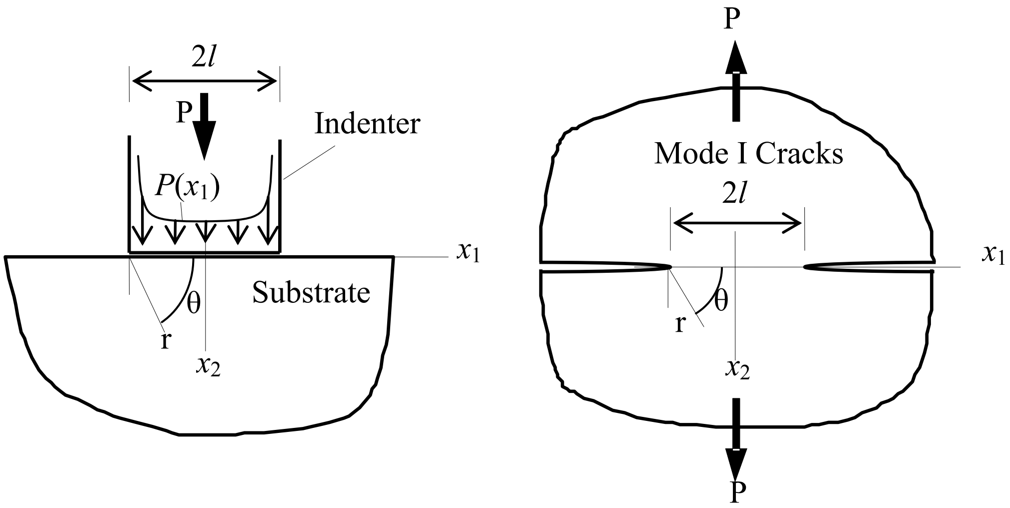

Fig.1 A schematic representation of the indentation. (a) Contact by a rigid flat-end indenter. (b) Related Mode I crack.

Interestingly, if the negative sign in Eq. (3) is changed into positive, this result will be the same as that for a plane semi-infinite crack (with an uncracked ligament of 2*l*) loaded remotely with a normal load *P*. In fact, for any Mode I indentation configuration, the related crack model can be found similar to the Fig.1(a) and (b). This finding is important as it shows that there is a K-dominant region next to the corner of the indenter in the indentation case. Then the contact failure can be treated as a crack growth problem with a well-defined stress singularity. As shown in Eq. (3), the indentation stress intensity factor $K_{I\text{-}ind}$ is the only parameter controlling the stress field, which means that the boundary cracking along the contact edge should be controlled by the same mechanism of fracture similar to the Mode I crack.

For the layered substrate as shown in Fig. 3, the singular stress field within thin the coating layer next to indenter corners can be given also by the form of the Eq.(3). In this case, the ISIF should be determined by the proposed method in this work.

3. CONSERVATION INTEGRAL

For the two-dimensional elastic solids, the following integrals can be given from the conservation law (Eshelby 1951, Sih 1969, Budiansky and Rice 1973).

$$J_j = \int_s \left(w n_j - T_i u_{i,j} \right) ds \tag{5}$$

Eq.(5) has two components, J_1 and J_2. For a closed integration path without any crack and cavity in it, J_i will vanish. J_1 is the so-called J-integral and J_2 the G^*-integral (Xie et la. 1998) . They can all be used to calculate the stress intensity factors for the cracked elastic bodies. In the following sections some key steps to calculate the ISIF for the indentation have been developed based on the J_1-integral.

4. PATH INDEPENDENT FOR HOMOGENOUS SUBSTRATE

For a closed integration path $s_{afedcba}$ as shown in Fig.2, following result can be given.

$$J_1 = \oint_{s_{afedcba}} (wn_1 - T_i u_{i,1}) ds = 0 . \quad (6)$$

If the path $s_{afedcba}$ is divided into $s_{afedcba} = s_{ab} + s_{bcd} + s_{de} + s_{afe}$, because of $n_1 = 0$ on surface of the substrate, $T_i = 0$ on the s_{ab}, $T_1 = 0$ and $u_{2,1} = 0$ on the s_{de}, we have

$$J_1 = \int_{s_{ab}} (wn_1 - T_i u_{i,1}) ds = 0 \quad (7)$$

and

$$J_1 = \int_{s_{de}} (wn_1 - T_i u_{i,1}) ds = 0 . \quad (8)$$

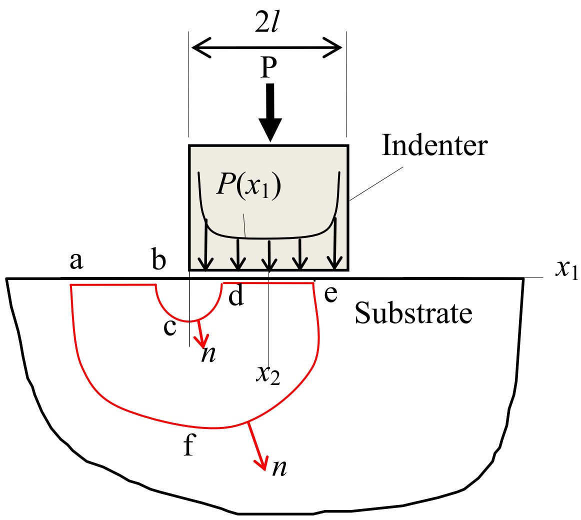

Fig.2 Integration path for homogenous substrate.

Then, from Eq.(6) it following that

$$J_1 = \oint_{s_{afedcba}} (wn_1 - T_i u_{i,1}) ds = \int_{s_{afe}} (wn_1 - T_i u_{i,1}) ds - \int_{s_{bcd}} (wn_1 - T_i u_{i,1}) ds = 0 . \quad (9)$$

It can then be rearranged to give

$$J_1 = \int_{s_{afe}} (wn_1 - T_i u_{i,1}) ds = \int_{s_{bcd}} (wn_1 - T_i u_{i,1}) ds , \quad (10)$$

which means that along any two paths, such as s_{afe} and s_{bcd}, starting from the any point on the left free boundary to any point within the contact boundary, the results of J_1-integral are identical. It shows theoretically that the integral is path independent.

If the integration path s_{bcd} is half circle and within the K-dominant region, it is not difficult to get

$$J_1 = \int_{s_{bcd}} (wn_1 - T_i u_{i,1}) ds = \frac{1-\mu^2}{2E} K_{I-ind}^2 \text{ (plane strain).} \tag{11}$$

Then, Eq.(10) becomes

$$J_1 = \frac{1-\mu^2}{2E} K_{I-ind}^2 = \int_{s_{afe}} (wn_1 - T_i u_{i,1}) ds \,. \tag{12}$$

This equation is a key formula to construct a method to calculate the ISIFs induced by the indentation. This equation can be applied to indentation problems with any infinite and finite boundaries.

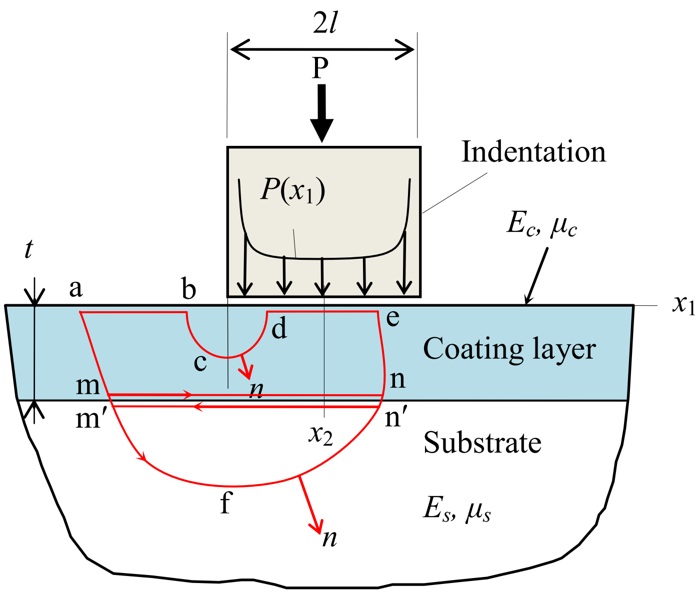

Fig.3 Indentation path for the layered substrate

5. PATH INDEPENDENT FOR LAYERED SUBSTRATE

For the layered substrate, two closed integration paths as shown in Fig.3 will be considered in present work. One is the path $s_1 = s_{mn} + s_{nedcbam}$ and the other $s_2 = s_{m'fn'} + s_{n'm'}$. Additionally, as $n_1 = 0$, T_i and $u_{i,1}$ are continuous on the paths s_{mn} and $s_{n'm'}$, from conservation law, we have the following contour integrals.

$$J_1 = \int_{s_{mn}} T_i u_{i,1} ds + \int_{s_{nedcbam}} (wn_1 - T_i u_{i,1}) ds = 0 \text{ for the closed path } s_1, \tag{13}$$

$$J_1 = \int_{s_{m'n'}} T_i u_{i,1} ds + \int_{s_{m'fn'}} (wn_1 - T_i u_{i,1}) ds = 0 \text{ for the closed } s_2 \tag{14}$$

and
$$J_1 = \int_{s_{m'n'}} T_i u_{i,1} ds = -\int_{s_{mn}} T_i u_{i,1} ds \tag{15}$$

According to Eqs.(7), (8), (13)-(15), it is not difficult to get

$$J_1 = \int_{s_{bcd}} (wn_1 - T_i u_{i,1}) ds = \int_{s_{afe}} (wn_1 - T_i u_{i,1}) ds , \tag{16}$$

which indicates that the integral is path independent for composite substrate similar to the Eq.10. If the integration path s_{bcd} is half circle and within the K-dominant region, the following equation can be found.

$$J_1 = \frac{1-\mu_c^2}{2E_c} K_{I-ind}^2 = \int_{s_{afe}} (wn_1 - T_i u_{i,1}) ds . \tag{17}$$

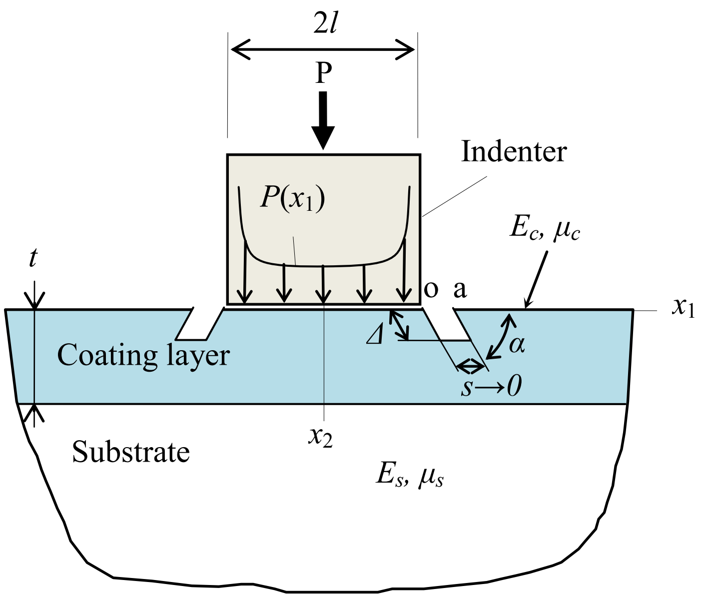

Fig.4 Boundary translation and crack embryo.

6. CRITICAL FRACTURE CONDITION BENEATH THE BEARING SURFACE

For an elastic substrate, the energy released rate of boundary cracking for indentation can be defined (Xie and Hills 2007).

$$G = -\frac{\partial \Pi}{\partial \Delta} = \int_{s \to 0} w e_i n_i ds . \tag{18}$$

Now, let all points on the boundary *s* as shown in Fig.4 move in the same direction, and combine this result with the conservation law J_i for two-dimensional problem. The energy release rate expressed by Eq.(18) can then be arranged to give

$$G = (J_1)|_{s\to 0} \cos\alpha + (J_2)|_{s\to 0} \sin\alpha , \tag{19}$$

where $(J_1)|_{s\to 0}$ denotes the driving force of boundary cracking in direction x_1 and $(J_2)|_{s\to 0}$ denotes the driving force in direction x_2, when the limits taken exist; J_1 and J_2 can be given by Eq.(5). Thus, the energy needed to start a new crack at the edge of the indenter has been found, by evaluating the contour integrals along the paths shown in Fig. 5.

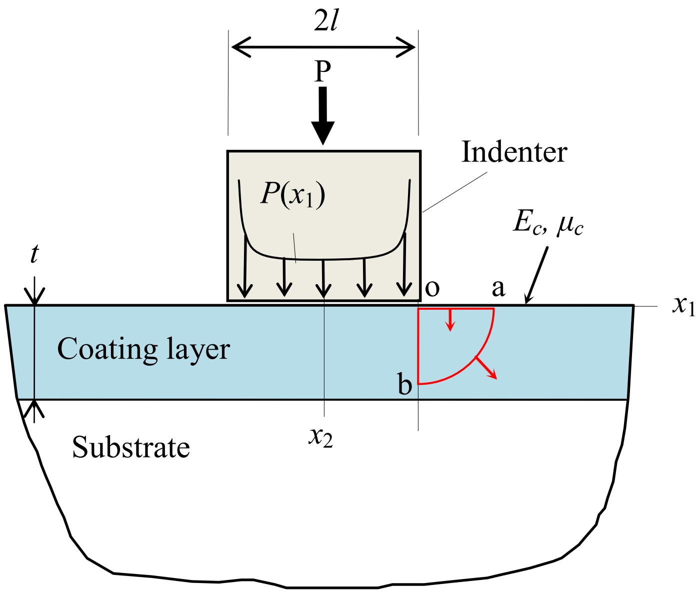

Fig.5. Contour integral for J2.

First, the value of J_1 was found. Let $s = s_{oa}$ in Eq.(5) as shown in Fig.5 adjacent to right corner of the indentation. Because of the $T_i = 0$ and $n_1 = 0$ on the integration path s_{oa}, then J_1 in this case can be found as

$$J_1 = \int_{s_{ao}} (wn_1 - T_i u_{i,1}) ds = 0 . \tag{20}$$

We turn, now, to an evaluation of J_2. Let $s = s_{ao}$ in Eq.(5) and take the limit $s_{oa} \to 0$, where s_{oa} and s_{oba} form a closed contour as shown in Fig.5. In this case, the J_2 integral becomes

$$J_2 = \int_{s_{oa}} (wn_2 - T_i u_{i,2}) ds = \int_{s_{ob}} (wn_2 - T_i u_{i,2}) ds + \int_{s_{ba}} (wn_2 - T_i u_{i,2}) ds . \tag{21}$$

Note that s_{ob} is a straight line and s_{ba} is a quarter of a circle. Along these two paths, the following integration results have been obtained (Xie et al. 1998).

$$\int_{s_{ob}} (wn_2 - T_i u_{i,2}) ds = 0 \tag{22}$$

and

$$\int_{s_{ba}} (wn_2 - T_i u_{i,2}) ds = \frac{(1-\mu_c^2) K_{I-ind}^2}{2\pi E_c} , \text{ plane strain.} \tag{23}$$

Thus, Eq. (21) now becomes

$$J_2 \left|_{s_{oa}\to 0} = \frac{\left(1-\mu_c^2\right)K_{I-ind}^2}{2\pi E_c}\right. . \tag{24}$$

From Eq.(19), the total energy release rate of boundary cracking at any angle α can be found as

$$G = \frac{\left(1-\mu_c^2\right)K_{I-ind}^2}{2\pi E_c}\sin\alpha . \tag{25}$$

The energy release rate is now maximised with respect to α by setting $\frac{dG}{d\alpha}=0$, i.e., $\cos\alpha = 0$, which gives

$$\alpha_c = \frac{\pi}{2}, \tag{26}$$

where α_c are the possible cracking angles at which G exhibits a maximum value, and explicitly this is given by

$$G_{\max} = \frac{\left(1-\mu_c^2\right)K_{I-ind}^2}{2\pi E_c} . \tag{27}$$

The most widely used criterion for assessing the extension of a crack is Griffith's (1921). His theory involves an energy balance, which states that the energy used in creating a new fracture surface from a solid body must be supplied from a combination of released elastic strain energy and from any work done by the applied load. The energy criterion of Griffith yields, in principle, a minimum critical load for failure. Thus, the critical condition for cracking of the boundary of a quasibrittle coating layer is given by

$$G_{\max} = G_c . \tag{28}$$

For a standard cracked specimen subjected to Mode I loading, the critical value of energy release rate can be given by

$$G_c = J_{IC} = K_{IC}^2\left(1-\mu_c^2\right)/E_c , \tag{29}$$

where K_{IC} is the fracture toughness of coating layer. Thus, according to Eqs. (27)-(29), the critical condition for indentation cracking of the boundary is found as

$$K_{I\text{-}ind} = \sqrt{2\pi}K_{IC} . \tag{30}$$

Eq. (30) gives a K-based critical condition, which defines a relationship between the ISIF and fracture toughness for boundary cracking of the coating layer. It should be emphasized that, in all of the above, the ISIF being employed is that defined by Eq. (17), and relates to the stress intensity prevalent at the indenter corners. Eq.(30) can be used to calculate the critical load to cause initiate fracture of the contact boundary.

7. Numerical example

A numerical example for the layered substrate under indentation is considered in this section. The mechanical properties of the materials for the substrate and coating layer are given in the table 1. The normalized ISIF have been gotten by using the Eq.(17) and the finite element method (FEM). In the FEM analysis, ANSYS10 Finite Element Package had been used and the eight-node plane structural element and concerned contact element have been selected. Around the indenter corners, we used a highly refined mesh where the tip was surrounded by six-node triangular quarter-point elements. More than 10,000 nodes and 7000 elements had been used for meshing the coating layer and substrate. Fig.6 gives schematically the relationship between the ISIF, geometrical and mechanical parameters. Additionally, from Eqs.(17) and (30) the critical condition in the form of schematics can be found in Fig.7. It shows that when l/t is smaller, the stress intensity factors are weakly dependent on the E_c / E_s. In this case, the ISIF is close to one expressed by Eq.(4), which gives the following result.

$$P_c = \pi\sqrt{2l}K_{IC} . \tag{31}$$

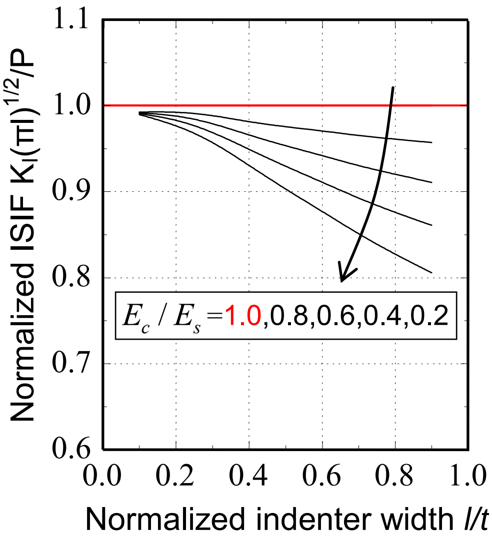

Fig.6 Normalized stress intensity factors for different parameters

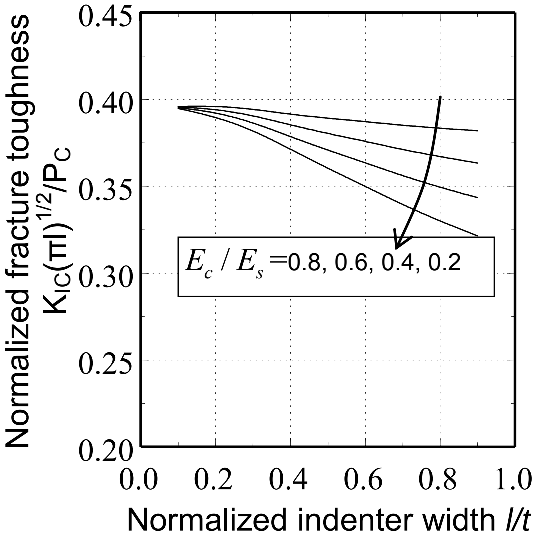

Fig.7 Critical condition for different parameters

8. CONCLUSIONS AND REMARKS

When a rigid flat-ended indenter is pressed into half-plane layered substrate, a singular stress field and K-dominant region will arise. It may lead fracture of the coating layer at a characteristic load. The ISIF $K_{I\text{-}ind}$ is the only parameter controlling the stress field next to the corner of the indenter, which represents stress intensification similar to the Mode I crack. For the half-plane homogenous substrate, it is not difficult to get the exact solution of the ISIFs (Nadai 1963). However, for the layered and multi-layered engineering structures under indentation, it is nearly impossible to get the exact solutions of the ISIFs. It is positive and significant to develop a method, which can be applied both to infinite and finite boundary indentation. In present article, a new method to formulize the ISIFs for the indentation is proposed based on the conservation integral. The main specialties of the proposed method and the critical condition of the boundary cracking are investigated. Some numerical examples have been given for the layered substrate under indentation.

Additionally, the some strength-related mechanical properties of materials in the state of coating, such as the fracture toughness, have been attracting lot of attentions in the recent years. It is nearly impossible to test the fracture toughness in the micro- and nano-scale by using the traditional pre-cracked specimen and concerned method. Unlike the traditional test method, the current study shows that the singular stress field and the K-dominant region can be induced by the indentation on the flaw-free surface of the coating layer, which implies the potential possibility to develop a very simple and practical technique to test the fracture based on the proposed fracture theory for the coating layered materials by the indentation.

Table 1. Normalized ISIF and normalized width of the indenter

	l/t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Normalized SIF. Eq.(3) $\mu_c = 0.25$ $\mu_s = 0.3$ $E_s = 200GPa$	$E_c/E_s = 0.2$	0.989	0.977	0.958	0.930	0.902	0.877	0.850	0.826	0.805
	$E_c/E_s = 0.4$	0.990	0.984	0.969	0.948	0.929	0.911	0.892	0.875	0.860
	$E_c/E_s = 0.6$	0.991	0.988	0.979	0.965	0.953	0.942	0.930	0.920	0.910
	$E_c/E_s = 0.8$	0.992	0.993	0.988	0.981	0.975	0.970	0.965	0.960	0.957

Acknowledgements

This work was supported by National Natural Science Foundation of China (Grant Nos. 50771052 and 50971068, 11272141) and Natural Science Foundation of Liaoning (Grant Nos. LS2010100 and 20102129)

REFERENCES

Budiansky, B., Rice, J.R. (1973), “Conservation Laws and Energy-Release Rates,” *J. Appl.Mech., ASME*, **40**, 201-203.

Eshelby, J. D. (1951), “The force on an elastic singularity. Philos,”*Trans. Royal Soc. London.*, A **244**, 87-112.

Ford, C., Bush, M. B., Hu, X.Z., and Zhao, H. (2004), “Numerical interpretation of cone crack initiation trends in a brittle coating on a compliant substrate,” *Mat. Sci. Eng.*, A **380**, 137-142.

Frank, F. C., Lawn, B. R. (1967), “On the theory of Hertzian fracture,” *Proc. Royal Soc. London.*, A **299**, 291-306.

Griffith, A. A. (1921), “The phenomena of rupture and flow in solids,” *Philos. Trans. Royal Soc. London.*, A **221**, 163-198.

Nadai, A. I. (1963), “Theory of flow and fracture of solids (Vol.2),” McGraw-Hill, New York.

Porter, M.I., Hills, D.A. (2002), “Note on the complete contact between a flat rigid punch and an elastic layer attached to a dissimilar substrate,” *Int. J. of Mech. Sci.*, **44**, 509–520.

Sih, G. C. (1969), “Dynamic aspects of crack aspects of crack propagation,” In: Inelastic behaviour of solids. Mc-Graw-Hill Book, Co., New York, 607-639.

Xie,Y.J. Hills, D.A. (2008), “Quasibrittle fracture beneath a flat bearing surface,” *Eng. Fract. Mech.*, **75**, 1223-1230.

Xie, Y.J., Xu, H., Li, P.N. (1998), “Crack mouth widening energy-release rate and its application,” *Theor. Appl. Fract. Mech.*, **29**, 195-203.