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A Surrogate Model for a Type of Nonlinear Hysteretic System with Application to Wind Response Analysis

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ABSTRACT

The nonlinear response of civil structures is often characterized by the phenomenon of hysteresis. The ongoing research effort in relevant topics has been generating scores of analytical descriptions corresponding to a number of nonlinear hysteretic systems. Nevertheless, many conventional time integration procedures associated with these descriptions can be arduous. Focusing on a typical description, this paper presents the development of a surrogate model in place of the conventional procedures with the aim of improving the time integration efficiency. In particular, the performance of the surrogate model is demonstrated by studying the nonlinear response of the hysteretic system due to wind loads. Options for further performance improvement of the surrogate model, together with its potential application to safety assessment and health monitoring of structures, are also discussed.

1. INTRODUCTION

In the context of inelastic analysis of civil structures, the phenomenon of hysteresis is an important, perhaps inevitable issue that needs to be appropriately taken into account. The hysteresis phenomenon can be described by various plasticity models in which hysteresis manifests itself in the constitutive relations of the materials involved. Alternatively, the phenomenon may also be allowed for at the structural system level, as investigated by Mostaghel (1999) and Mostaghel and Byrd (2000), among other researchers. The relative effectiveness and convenience of these two kinds of description are highly problem-specific, and moreover it is worth noting that in some cases the boundary between them could blur.

Once the hysteresis description is selected and the structural equilibrium equation,

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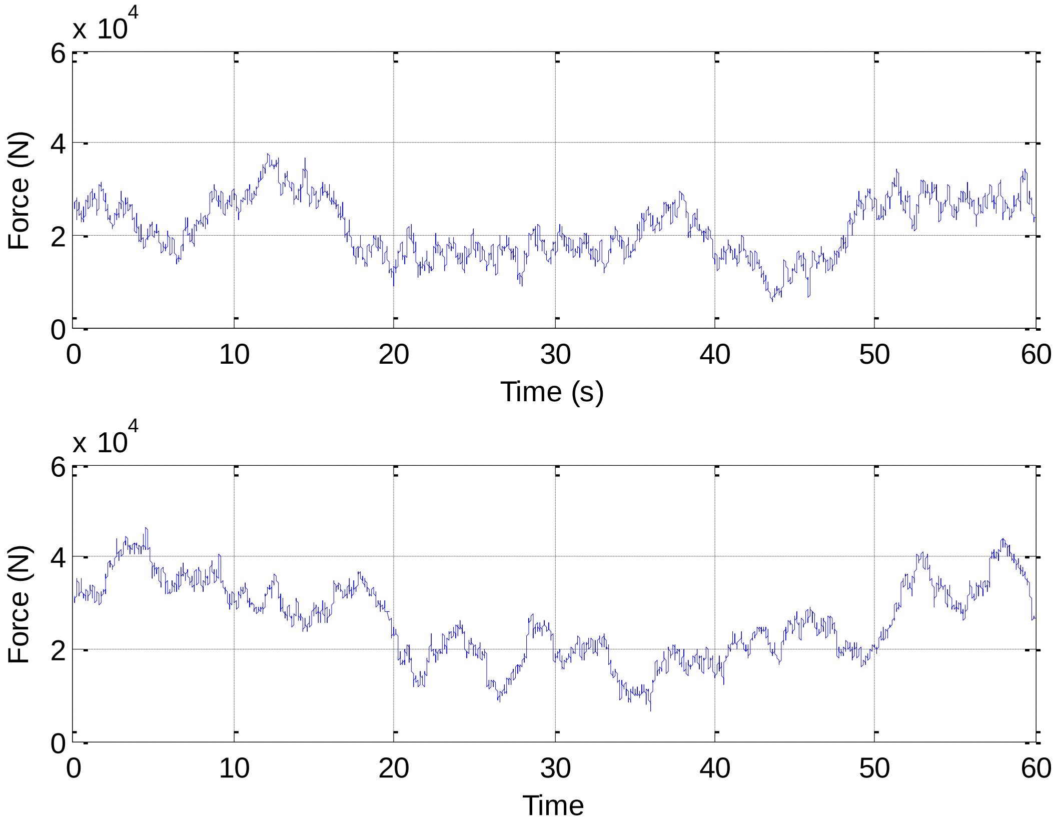

Fig. 1. Examples of the wind-induced excitation-force time histories at $h = 7.5$ m (upper subfigure) and $h = 15$ m (lower subfigure)

which could be static or dynamic, is set up, the structural response can then be computed. Conceptually this process sounds fairly natural and smooth, while practically it may result in prolonged computation time, which could at worst become a prohibitive inconvenience especially when the process is to serve as a subroutine to be frequently called in a larger algorithm. For instance, in a Monte Carlo simulation based reliability analysis, it is not uncommon for the process to be repeated thousands of times.

Strong wind loads may lead to significant economic and even life loss, as continually exhibited by numerous cyclones, hurricanes, and typhoons (Boughton et al. 2011). With strong wind loads identified as a kind of extreme load, this paper is concerned with the efficient wind response computation of a typical hysteretic structure. Specifically, a surrogate model is developed to replace some conventional time marching procedures.

In the next section the hysteretic system being studied is defined, followed by the construction and validation of a corresponding surrogate model. Then in the third section the surrogate model is applied to assess the safety performance of the system, and due consideration is given to the relevant data missingness events that could occur during structural appraisal activities in reality.

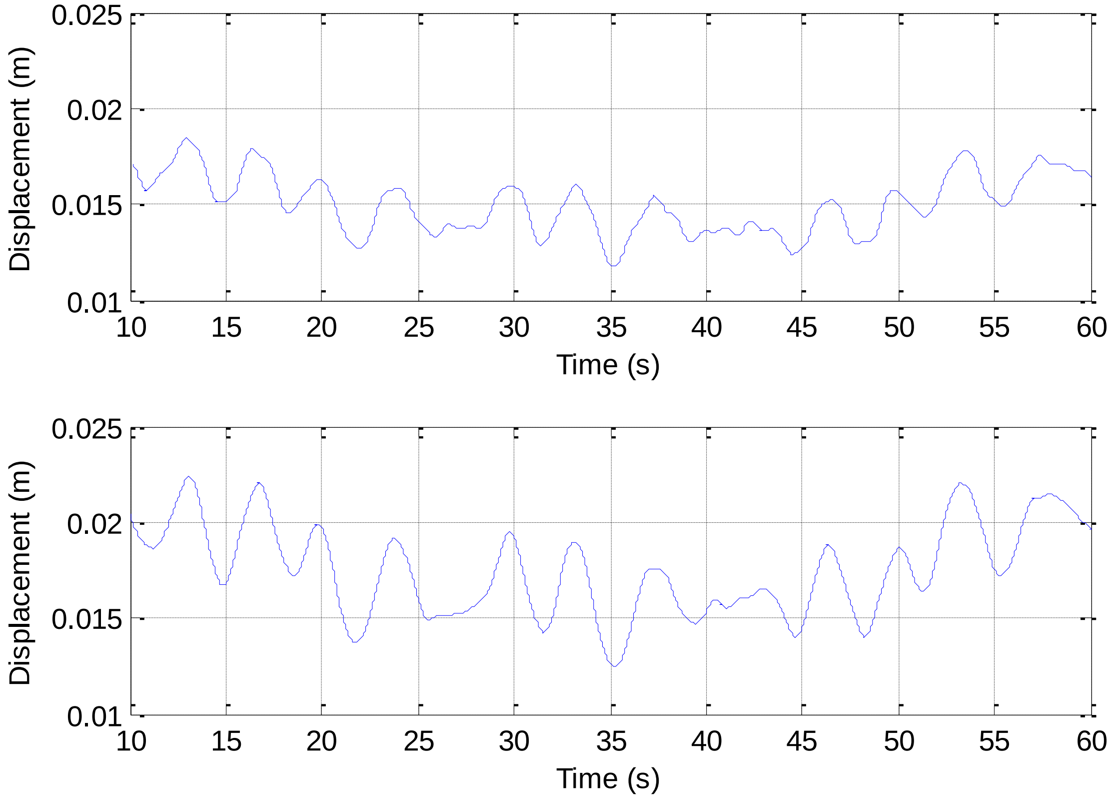

Fig. 2. Displacement time history at the first floor level (upper subfigure) and that at the roof level (lower subfigure) corresponding to the excitation forces shown in Fig. 1

2. SURROGATE-MODEL CONSTRUCTION AND VALIDATION

A two-story nonlinear hysteretic shear frame as described by Mostaghel (1999) and Mostaghel and Byrd (2000) is used in this study to illustrate the surrogate-model development. The equations of motion of the frame are shown in Eqs. (1) - (4):

$$\mathbf{M}\ddot{\mathbf{U}}(t)+\mathbf{C}\dot{\mathbf{U}}(t)+\gamma\mathbf{K}'\mathbf{U}(t)+(1-\gamma)\mathbf{K}''\mathbf{V}(t)=\mathbf{F}_w(t) \tag{1}$$

$$\dot{\mathbf{V}}(t)=\mathbf{G}\Big(\mathbf{U}(t),\ \mathbf{V}(t),\ \dot{\mathbf{U}}(t),\ \dot{\mathbf{V}}(t)\Big)\dot{\mathbf{U}}(t) \tag{2}$$

in which

$$\mathbf{K}'=\begin{pmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{pmatrix} \tag{3}$$

$$\mathbf{K}''=\begin{pmatrix} K_1 & -K_2 \\ 0 & K_2 \end{pmatrix} \tag{4}$$

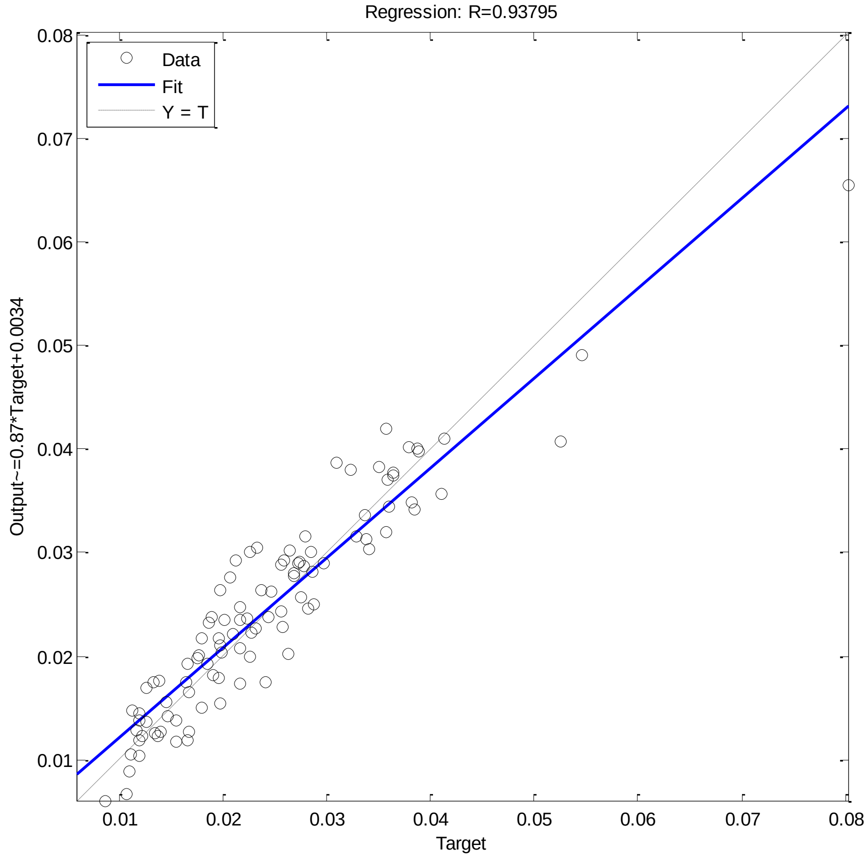

Fig. 3. Regression analysis of the maximum inter-story relative displacements obtained from the surrogate model on those from the time marching procedures

where t is the time; $\mathbf{M}$ is the lumped mass matrix; $\mathbf{C}$ is the damping matrix; $\mathbf{K}'$ and $\mathbf{K}''$ are the stiffness matrix and the auxiliary stiffness matrix, respectively; $\mathbf{F}_{\mathrm{w}}(t)$ is the wind-induced excitation-force time history; $\mathbf{U}(t)$ is the displacement time history; the parameter γ is known as the post-yield-to-pre-yield stiffness ratio; and K_1 and K_2 respectively denote the story stiffness for the first story and that for the second story. For the two-story shear frame being considered, the equations of motion contain a total of four unknown functions, i.e., the four components of $\mathbf{U}(t)$ and $\mathbf{V}(t)$. $\mathbf{G}$ is a function of $\mathbf{U}(t)$, $\mathbf{V}(t)$, $\dot{\mathbf{U}}(t)$, and $\dot{\mathbf{V}}(t)$, and its specific expression can be found in Mostaghel (1999) and Mostaghel and Byrd (2000).

The story stiffnesses K_1 and K_2 are assumed to have a bivariate normal distribution with both of the means being 1×10^{7} N/m, both of the variances being 2.25×10^{12} $\mathrm{N}^2/\mathrm{m}^2$, and a covariance of 1.125×10^{12} $\mathrm{N}^2/\mathrm{m}^2$. Using a uniform story height of 7.5 m, the wind-induced excitation force $\mathbf{F}_{\mathrm{w}}(t)$ can be simulated based on a representative wind load model as in Soong and Grigoriu (1993) and Simiu and Scanlan (1996). Fig. 1 illustrates

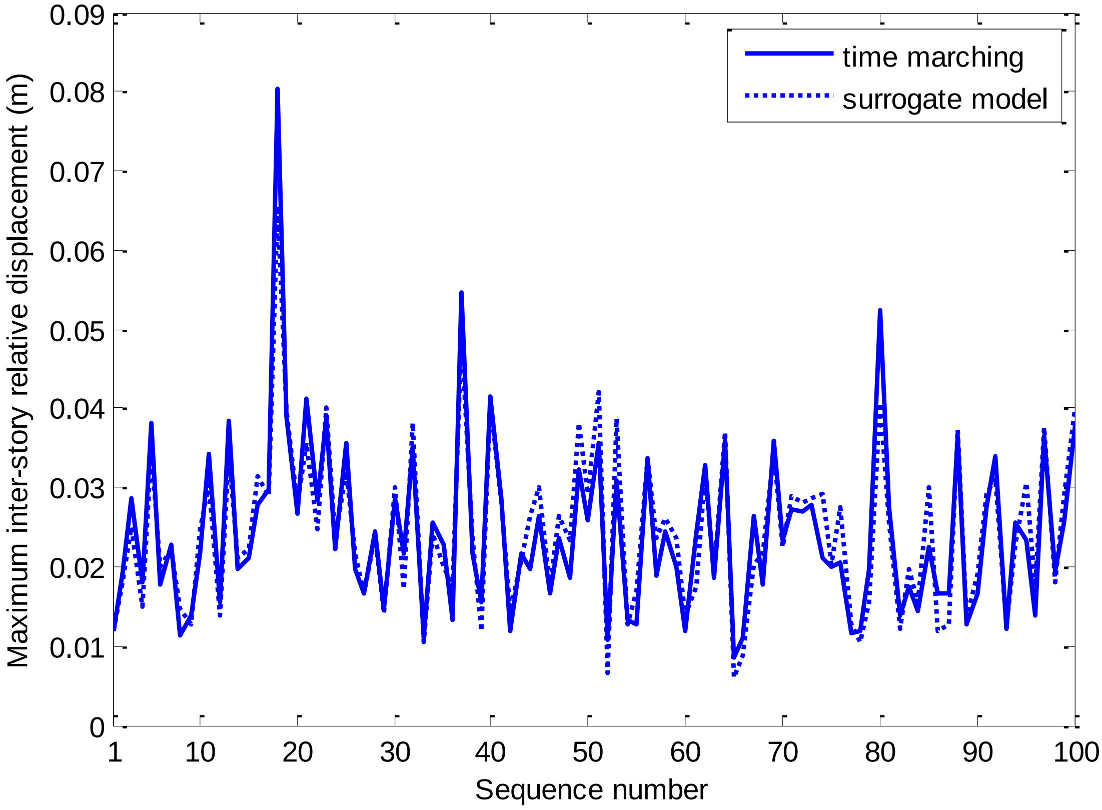

Fig. 4. Comparison between the maximum inter-story relative displacements resulting from the surrogate model and those from the time marching

Table 1. Structural appraisal data for K_1 and K_2 with some missing data points*

Story ID	Incomplete structural appraisal data for K_1 and K_2 ($\times 10^7$ N/m)
1	0.9790; 0.8685; 0.9251; 1.1092; 1.1283; 0.7588; NA; 0.9295; 1.0301; 1.0156; 1.0684; 1.0402; NA; NA; NA; 1.0641; NA; 0.9946; 1.0817; NA; 1.0843; 0.9329; 1.2423; 0.8932; 1.0120; 1.2727; 1.0925; NA; 0.8282; 1.1834.
2	1.2293; 0.9902; 0.7541; NA; NA; NA; 1.0702; 1.1302; NA; NA; 1.0836; 1.1869; 1.0253; 0.9788; 0.9552; NA; NA; NA; 0.9465; 0.9008; 1.1277; NA; NA; NA; NA; 0.8916; 1.2934; 1.2529; 1.0118; 0.9083.

* The missing data points are indicated by NAs.

the simulated $\mathbf{F}_w(t)$ for a height h of 7.5 m (i.e., the first floor level) or 15 m (i.e., the roof level). Corresponding to a set of $\mathbf{F}_w(t)$ data, some well-established time marching

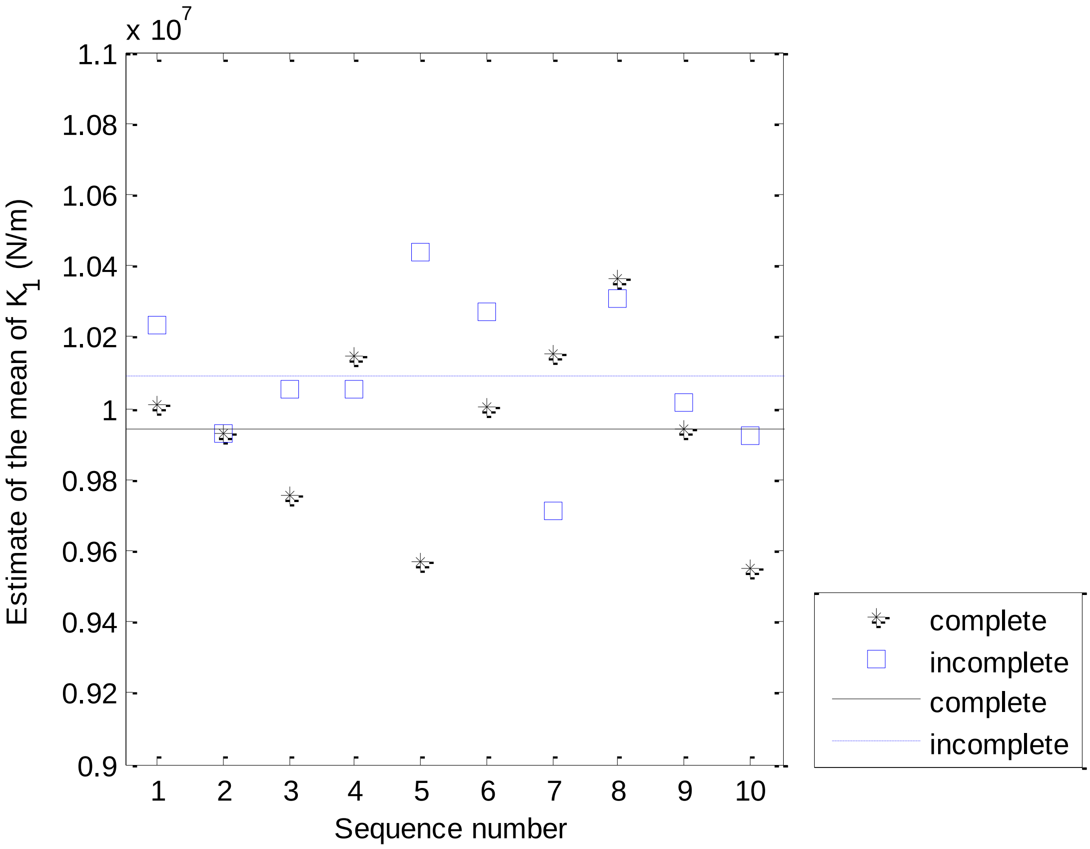

Fig. 5. Estimates of the mean of K_1 in the complete- and incomplete-data scenarios

Table 2. Estimated wind fragilities in the complete- (Case I) and incomplete-data (Case II) scenarios (threshold for the maximum inter-story relative displacement: 0.02 m)

Case ID	Estimated wind fragilities
I	0.623; 0.623; 0.659; 0.586; 0.682; 0.571; 0.569; 0.533; 0.638; 0.702.
II	0.567; 0.645; 0.595; 0.566; 0.507; 0.556; 0.680; 0.508; 0.579; 0.657.

procedures are readily available to compute the displacement time history $\mathbf{U}(t)$. For example, Fig. 2 shows the $\mathbf{U}(t)$ values obtained by using the classical fourth-order Runge-Kutta algorithm. The maximum inter-story relative displacement, an important quantity in structural safety assessment, can then be computed.

In order to construct a surrogate model to determine the maximum inter-story relative displacement in a more efficient way, a feedforward backpropogation neural network is created and trained. The database used for the neural network training is formed by 2,000 independent runs of the fourth-order Runge-Kutta algorithm, and for each run the time marching stops at $t = 60$ s. When training the neural network, each of the 2,000 sets of the input data sequentially comprises the two story stiffnesses, the sampled (sampling rate: 1/50) excitation-force time history at the first floor level, and

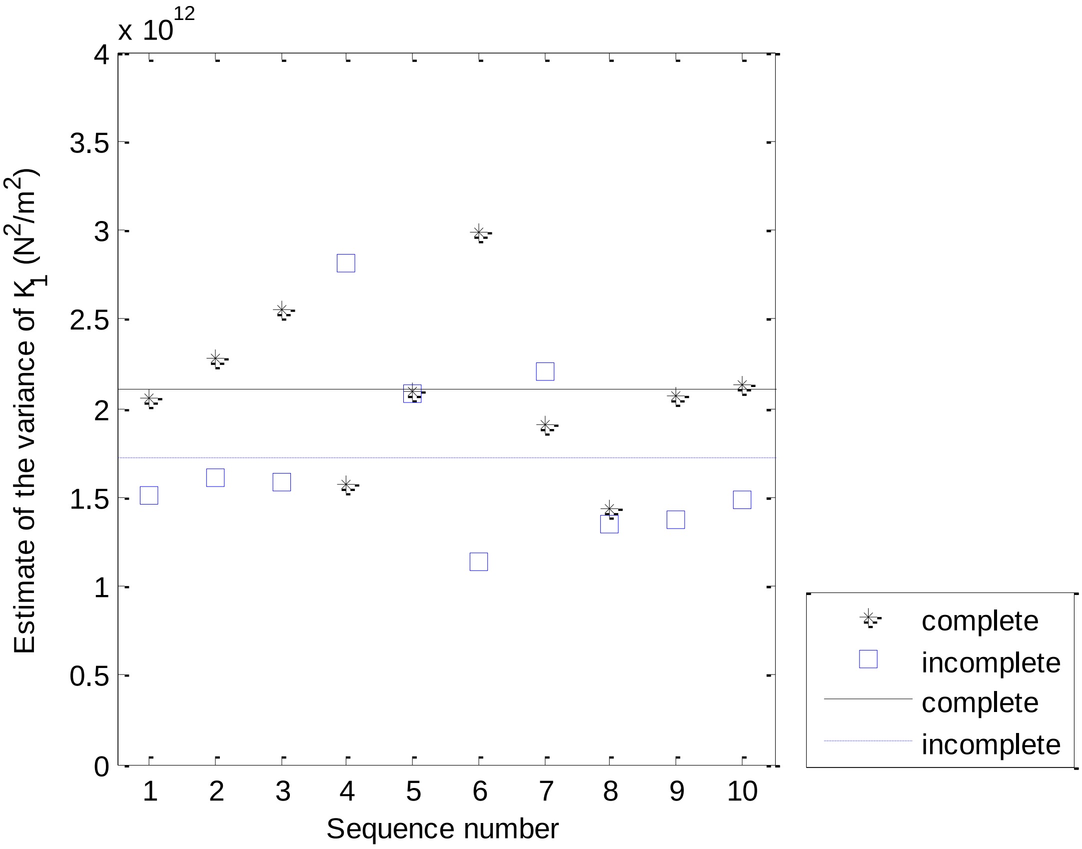

Fig. 6. Estimates of the variance of K_1 in the complete- and incomplete-data scenarios

that at the roof level, and each of the output data is the maximum inter-story relative displacement. With the fourth-order Runge-Kutta algorithm chosen as the benchmark time marching procedures, the resulting trained neural network is then independently validated, as in Figs. 3 and 4. It can be observed that the maximum inter-story relative displacement values yielded by the trained neural network agree with those from the time marching procedures reasonably well.

3. WIND SAFETY ASSESSMENT BY USING THE SURROGATE MODEL

The potential of the surrogate model in the areas of structural safety assessment and structural health monitoring is briefly explored through an example. The influence of the structural appraisal data missingness is allowed for as well (Wang et al. 2013).

Consider the two-story hysteretic shear frame in Section 2. Suppose that the frame has been in service for a period of time, and its in-situ story stiffnesses K_1 and K_2 need to be evaluated using structural appraisal techniques. Table 1 gives an example of some incomplete structural appraisal data, where it is assumed that the relevant appraisal data points are missing completely at random (Heitjan and Basu 1996). An algorithm known as the expectation-maximization algorithm (Dempster et al. 1977; Wu

1983; Meng and van Dyk 1997; Novo and Schafer 2012; R Core Team 2012) can be applied to deal with the appraisal data missingness. Along with the developed surrogate model, the wind safety of this in-service shear frame can then be assessed. Indeed, as in Figs. 5 and 6 and Table 2, the scheme is effective in the sense that, corresponding to the estimated wind fragilities in the constructed complete- and incomplete-data scenarios, no significant difference is signaled by the two-sample Kolmogorov-Smirnov test with a significance level of 0.05.

4. CONCLUDING REMARKS

This paper designs a surrogate model useful for improving the efficiency in the response computation of a typical hysteretic structure under wind loads. It is expected that models of this kind could be combined with other pertinent techniques to better assess the wind safety performance of a broad range of in-service civil structures.

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