The 2013 World Congress on
Advances in Structural Engineering and Mechanics (ASEM13)
Jeju, Korea, September 8-12, 2013

Numerical modelling of steel fibre reinforced concrete beams subjected to fatigue loading

*Ahsan Parvez[1) and Stephen James Foster[2)

[1), 2)] *School of Civil and Environmental Engineering, University of New South Wales, Sydney, Australia*

[1)] m.parvez@unsw.edu.au

ABSTRACT

Development of steel fibre reinforced concrete (SFRC) can contribute to the durable design of fatigue prone structures such as bridge deck slabs, bridge girders and offshore installations. A study was undertaken to evaluate the fatigue life of reinforced concrete (RC) beams with 0, 0.4 and 0.8 per cent, by volume, of steel fibres. It was observed that the fibres significantly enhance the fatigue strength over that of conventional RC with the increase in fatigue life as a function of fibre volume. Steel fibres also decrease the crack widths, deflections and increase the stiffness. A finite element (FE) model for the analysis of SFRC beams under fatigue loading was developed and validated using the experimental results of this study. The results obtained using the FE model compare favourably with the test data.

1. INTRODUCTION

The design of reinforced concrete structures subjected to cyclic loading such as bridge deck slabs, bridge girders and offshore installations, for example, necessitate the consideration of fatigue. Such structures typically experience millions of stress cycles during their service life (Tilly, 1979). A wide range of tests have been conducted to understand the fatigue phenomenon of reinforced concrete structural members (CEB, 1988 and Mallet, 1991). Fatigue life depends upon the cyclic stress range – the lower the stress range, the greater the number of cycles to failure (Schutz, 1996). Fatigue of concrete is a progressive process of microcrack initiation and propagation leading to macrocracks that grow to a stage that failure occurs (Tilly, 1985). In the case of a reinforcing bar, cyclic load causes microcracking that initiates a stress concentration at the bar surface. As the cyclic stress continues, the crack propagates leading to sudden

[1)] PhD Student
[2)] Professor

fracture. On the other hand, the performance of reinforced concrete depends on the composite interaction between the reinforcement bar and the concrete (Mallet, 1991). Under repeated cyclic loading concrete mechanical properties change; the strains in concrete increase permanently and its stiffness decreases (Holmen, 1982).

Cyclic loading can be detrimental to structural performance and assessment of the fatigue life of structures is an important design criterion. SFRC is a relatively new engineering material has been found to improve fatigue life of plain concrete and several of its properties, e.g. crack resistance, impact and wear resistance, toughness and ductility (Yin et al., 1988, Lee et al., 2004). The latest research result (Parvez and Foster, 2012) shows that the steel fibers prolonged the fatigue life in SFRC beams significantly.

Finite element analysis (FEA) is widely applied to concrete structures based on the use of the nonlinear behaviour of materials. RC members subjected to monotonic loading have been modeled extensively. However, limited models have been developed for RC members subjected to fatigue loading. CEB (1996) provides constitutive models and FE formulation for the RC elements under cyclic loading exclusively. Dobromil et al. (2010) developed a stress based model (S-N curve) for fatigue of concrete under tension and implemented into the FE program ATENA. The S-N criterion is translated into material damage, which is introduced into the material model on the basis of stress increments at each material point and the number of cycles. Loo et al. (2012) developed a constitutive model for the modeling of fatigue behaviour of the RC members and included in the FE program RECAP (Foster, 1992, Foster et al, 1996, Foster and Marti, 2003). The model was then verified against test data. The model of Loo et al. (2012) can predict the fatigue life of RC members with reasonable level of confidence. However, these models need modification for using in SFRC members under fatigue.

For describing the constitutive behaviour of SFRC, a number of models have been proposed; the Variable Engagement Model (VEM) for Mode I fracture for SFRC is found to be simple and reliable and is used as the base material model for this study (Voo and Foster, 2004, 2009 and Foster et al.,2006). The VEM was incorporated in the fatigue model of Loo et al. (2012) for RC and a FEA was conducted using the program RECAP to predict the fatigue life and behaviour of SFRC beams.This paper reports the development of an FE model for the analysis of SFRC beams subjected to fatigue loading. The model was then verified with the available test data.

2. MATERIAL CONSTITUTIVES MODELS

In the FE modeling of RC, concrete can be modeled as two dimensional orthotropic membrane elements. The current FE formulation of RECAP is based on the cracked membrane model of Kaufmann and Marti (1998). Details of the material law can be found in Foster (1992), Foster et al. (1996) and Foster and Marti (2003). Voo and Foster (2004, 2009) incorporated VEM as a constitutive model for the FE modeling of SFRC under monotonic loading. The work of Loo et al. (2012) includes a detail description of the plane-stress non-linear FE formulation for RC elements under fatigue loading, are utilized herein. The precedence sections describe the various constitutive models adopted in this study for FE modeling of SFRC under fatigue loading.

2.1 Concrete in Compression

The stress-strain law of the concrete in compression under fatigue loading adopted was that of Holmen (1982). During cyclic loading the stress-strain response of concrete varies with the number of load cycles in three stages: (i) a rapid increase from 0 to about 10 percent of the fatigue life; (ii) a uniform increase from 10 percent to approximately 80 percent of the fatigue life; and (iii) a rapid increase to failure. Holmen described the first and second stage by the following relationship:

$$\text{Stage I:} \quad \varepsilon_{max} = \frac{1}{E_{sec}}\left|S_{max} + 3.18(1.183 - S_{max})(N/N_f)^{0.5}\right| + 0.413x10^{-3}S_c^{1.184}\ln(t+1) \quad (1)$$

$$\text{Stage II:} \quad \varepsilon_{max} = \frac{1.11}{E_{sec}}\left|1 + 0.677(N/N_f)\right| + 0.413x10^{-3}S_c^{1.184}\ln(t+1) \quad (2)$$

where ε_{max} is the maximum strain. E_{sec} is the secant modulus at the first cycle, S_{max} is the ratio of maximum stress to concrete strength; S_c is the characteristics stress level and is given as $S_m + RMS$, S_m is the mean stress ratio and is equal to $0.5(S_{min} + S_{max})$; S_{min} is the ratio of minimum stress to concrete strength; N is the number of load cycles; N_f is the number of load cycles to failure for a specified probability of failure; t is the duration of the alternating load (in hours), RMS is the root mean square value of the stress ratio. For sinusoidal loading, $RMS = (S_{min} + S_{max})/8^{0.5}$.

The secant modulus after the first cycle is reduced throughout the fatigue life according to S_{max}. For the current cycle number, N, the secant modulus E_N and reduces significantly with increasing cycle numbers, with the greatest reduction occurring before $0.2N_f$, where N_f is the number of cycles to failure. From $0.2N_f$ to $0.8N_f$, the modulus continues to decrease slowly until again decreasing at an increasing rate to failure. After failure of the concrete, ε_{max} is maintained as a constant corresponding to the value at failure and the secant modulus is taken as $E_N = (1 - 0.4\,N/N_f)E_c$, where E_c is the initial elastic modulus of concrete and noting that post-failure $N > N_f$.

2.2 Failure Cycles for Concrete in Compression

The CEB-FIP (1993) model is used to estimate the failure cycles, N_f of concrete under a constant amplitude loading. In compression, and for the minimum stress, S_{min}, in the range $0 \leq S_{min} \leq 0.8$, the failure cycle is given by

$$\log N_1 = (12 + 16S_{min} + 8S_{min}^2)(1 - S_{max}) \quad (3)$$

$$\log N_2 = 0.2\log N_1(\log N_1 - 1) \quad (4)$$

$$\log N_3 = \log N_2\,(0.3 - 0.375S_{min})/\Delta S \quad (5)$$

$$\text{If } \log N_1 \leq 6, \quad \text{then } \log N = \log N_1$$

$$\text{If } \log N_1 > 6 \text{ and } \Delta S \geq 0.3 - 0.375S_{min}, \quad \text{then } \log N = \log N_2$$

$$\text{If } \log N_1 > 6 \text{ and } \Delta S < 0.3 - 0.375S_{min}, \quad \text{then } \log N = \log N_3$$

where N_1, N_2 and N_3 represents the failure cycle envelope and $\Delta S = |S_{max}| - |S_{min}|$.

2.3 Concrete in Tension

The stress-strain relationship of SFRC in tension is modeled using VEM. The constitutive law of SFRC composed of the superposition of two components: (i) the tension softening law of the matrix without fibres; and (ii) contribution of the fibres after cracking of the matrix. For concrete without fibre, the bilinear stress-strain model of Petersson (1981) is used as shown in Fig.1a with the following tension softening parameter:

$$\alpha_1 = \frac{1}{3};\ \alpha_2 = \frac{2}{9}\alpha_3 + \alpha_1;\ \ \alpha_3 = \frac{18}{5}\frac{E_c G_f}{l_{ch}\ f_{ct}^2} \tag{6}$$

where E_c is the initial elastic modulus of concrete, G_f is the fracture energy of concrete matrix, l_{ch} is the characteristic length of the finite element and f_{ct} is the tensile strength of concrete.

For the strength contribution of the fibres, the stress-COD (Fig.1b) of the VEM is given by following equations (Voo and Foster, 2009):

$$\sigma_t = K_f\, \alpha_f\, \rho_f \tau_b \tag{7}$$

$$K_f = \frac{\tan^{-1}(w/\alpha)}{\pi}\left[1 - \frac{2w}{l_f}\right]^2;\ \ \text{for}\ \ l_f < l_c \tag{8}$$

$$K_f = \frac{4}{\pi l f^2}.\int_0^{\theta_{crit}}\left\{\max\left(l_{a,crit} - w, 0\right)\right\}^2 d\theta;\ \ \ \text{for}\ \ l_f \geq l_c \tag{9}$$

where σ_t is the tensile strength provided by steel fibre at the current crack width (w); K_f is the global orientation factor; α_f is the fibre aspect ratio ($\alpha_f = l_f/d_f$); ρ_f is the fibre volumetric ratio; τ_b is the average bond shear strength at the fibre and matrix interface; l_f is the fibre length; d_f is the fibre diameter; l_c ($l_c = 0.5 d_f\ \sigma_{fu}/\tau_b$) is the fibre critical length; σ_{fu} is the fibre ultimate tensile strength; α is the engagement parameter; and $l_{a,crit}$ is the critical fibre embedment length for fracture.

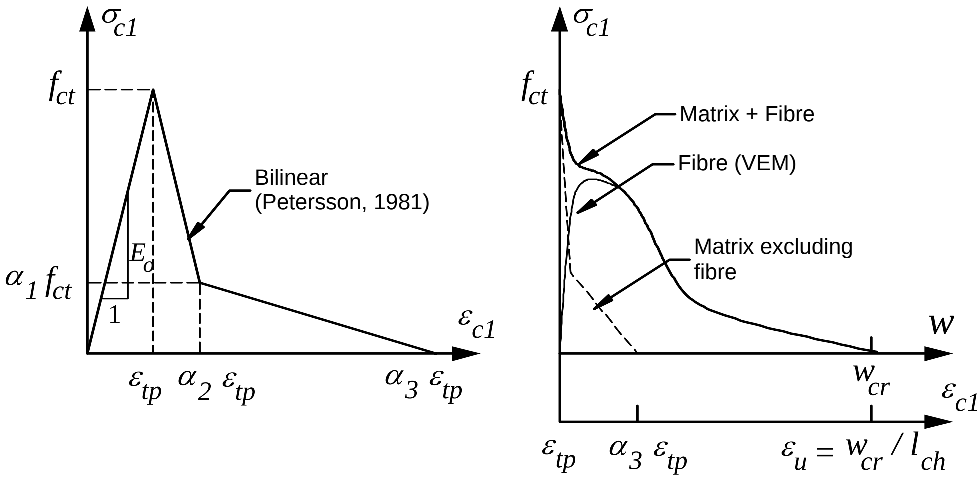
2.4 Reinforcing Bar in Fatigue

The failure cycle, N_f, of a reinforcing bar under a constant amplitude fatigue load is estimated based on the model of CEB-FIP (1993) given as:

$$\text{If}\ \Delta\sigma\ \leq\ \Delta\sigma_{N*},\ \log N_f = \log(N^*) - k_1(\log\Delta\sigma - \log\Delta\sigma_{N*}) \tag{10}$$

$$\text{If}\ \Delta\sigma\ \geq\ \Delta\sigma_{N*},\ \log N_f = \log(N^*) + k_2(\log\Delta\sigma_{N*} - \log\Delta\sigma) \tag{11}$$

where $\Delta\sigma$ is the stress range in the steel, $\Delta\sigma_{N*}$ is the stress range at N^* cycles and k_1 and k_2 are the stress exponents that depends on the bar size, and are given in the CEB-FIP (1993). The progressive increasing stress in the reinforcing bar was accounted for in determining the cycles to failure by introducing Palmgren-Miner rule



(a) (b)

Fig.1: Fibre reinforced concrete in tension: (a) bilinear stress-strain model for concrete matrix (Petersson, 1981) (b) variable engagement model (Voo and Foster, 2009)

(Miner, 1945). Fatigue failure does not occur where:

$$\sum_{\mathrm{i}=1}^{\mathrm{k}} \frac{\mathrm{N_i}}{\mathrm{N_{fi}}} < D \tag{12}$$

where $\mathrm{N_i}$ is the number of cycles of a particular stress range spectrum, $\mathrm{N_{fi}}$ is the number of cycles to failure for the given spectrum, k is the number of stress range spectrums and D is a damage parameter and is taken as D = 1. For the case, in which the stress in the reinforcing steel is less than the yield, the modulus of elasticity of the bars are assumed to be equal to the initial modulus ($\mathrm{E_s}$) for all cycles.

3. NUMERICAL EXAMPLES

To verify the FE model the results from the latest fatigue tests under taken at the University of New South Wales (UNSW) has been selected. The experimental program consisted of sixteen reinforced concrete beams that were prepared in two series with eight beams in each series. Each series were further subdivided in four groups. In each group of two beams a different volume fraction of steel fibres were used; namely, 0, 0.4 and 0.8 percent by volume. The fibres used were Dramix RC-65/35 (aspect ratio = 65 and length = 35 mm) and Dramix RC-80/60 (aspect ratio =80 and length = 60 mm) for Series 1 and Series 2, respectively. The details of the specimens are given in Fig. 2. The beams were simply supported with spans of 1200 mm for Series 1 and 3000 mm for Series 2. The beams were subjected to four-point loading with constant moment zone of 400 mm and 800 mm for Series 1 and Series 2, respectively. Six beams from

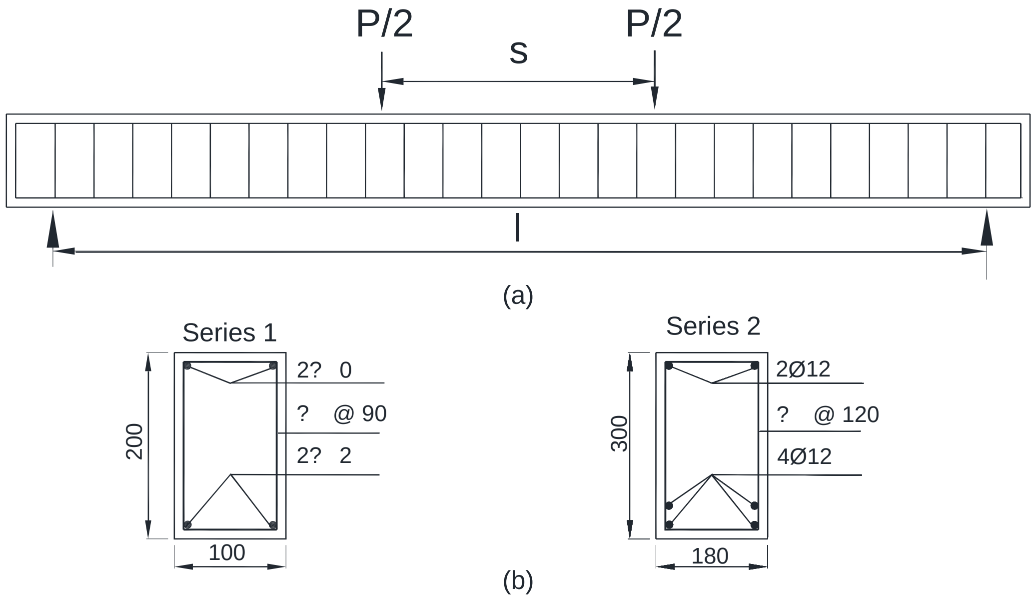


Fig.2: Details of test specimens: (a) elevation (b) section

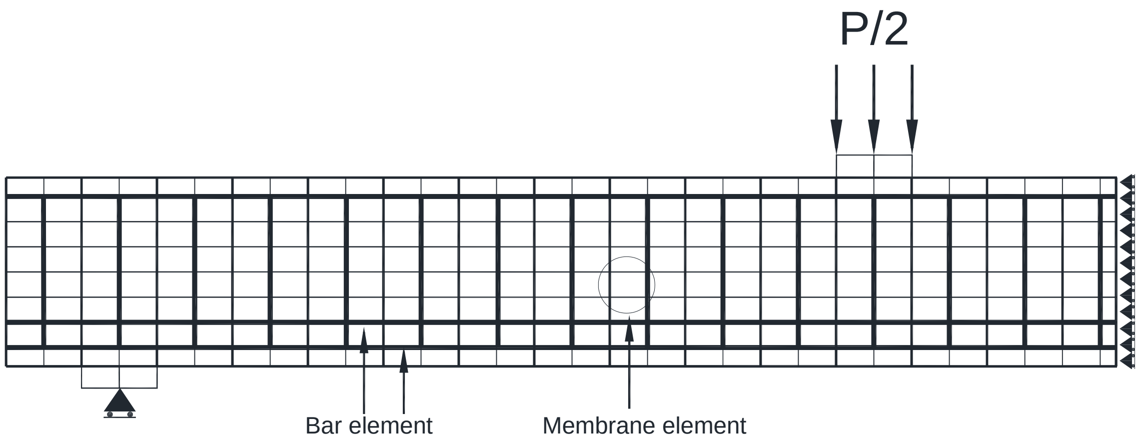


Fig.3: FE mesh for SFRC beams

each series (with fibre volume 0, 0.4 and 0.8 percent) were tested under fatigue loading. The load ranges for fatigue tests were selected based on the failure load observed in static tests. The load ranges were between 20 to 74 percent (22 to 78 kN) and 20 to 70 percent (21 to 74 kN) of static failure load for Series 1 and Series 2, respectively. The details of experiments and results of Series 1 can be found at Parvez and Foster (2012).

All beams were modelled using four-node isoparametric concrete elements and 4 by

Table 1- Material properties of the SFRC used in the FE modeling

Specimen Notation	**Series 1**			**Series 2**		
	1B	**1C**	**1D**	**2B**	**2C**	**2D**
Concrete						
f_{cm} (MPa)	52	55	61	39	35	38
f'_c (MPa)	47	50	56	36	32	35
f_{ct} (MPa)	2.47	2.55	2.70	2.2	2.0	2.1
E_c (GPa)	33.41	33.96	35.02	29.95	28.38	29.57
G_f (N/m)	130			155		
l_{ch} (mm)	50	50	50	50	50	50
ε_c	0.003	0.003	0.003	0.003	0.003	0.003
v	0.2	0.2	0.2	0.2	0.2	0.2
Reinforcing Steel						
Bar	2x12 mmϕ	2x12 mmϕ	2x12 mmϕ	4x12 mmϕ	4x12 mmϕ	4x12 mmϕ
Area (mm^2)	220	220	220	440	440	440
E_s (GPa)	200	200	200	200	200	200
E_w (MPa)	1300	1300	1300	1300	1300	1300
ε_y	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028
f_{sy} (MPa)	560	560	560	560	560	560
Steel Fibre						
ρ_f (%)		0.4	0.8		0.4	0.8
l_f (mm)		35	35		60	60
d_f (mm)		0.55	0.55		0.75	0.75
α_f		65	65		80	80
σ_{fu} (MPa)		1100	1100		1050	1050
T_b (MPa)		6.37	6.74		5.07	5.30
α-parameter		0.16	0.16		0.21	0.21
$\Delta\sigma_{N*}$ (MPa)	225	225	225	280	295	330

four-node stiff elements for the steel plates at the load points and supports. The steel reinforcement was modelled as two node bar elements. One half of the specimen was modeled accounting for symmetry. Details of the FE mesh used are given in Fig.3 and the material properties used for the FE models are given in Table 1. The material properties are obtained from respective cylinder and prism test. The tensile strength has been taken $f_{ct} = 0.36\sqrt{f_c'}$ as given in AS 3600 (2009). The average bond shear strength and engagement parameter in VEM is taken as, $\tau_b = 2.5 f_{ct}$ and, $\alpha = \frac{d_f}{3.5}$, respectively, as suggested by Voo and Foster (2009).

A load and cycle controlled solution scheme is used in FE analysis, as outlined in Loo et al. (2012). The state of the concrete at different cycle steps, for the stress range determined by the model response at that step, is taken into account by the FE model. The current stress condition for each individual concrete and steel are considered at each load step by the model. At the first cycle, the model is loaded in four equal increments until the load reaches the minimum load. The load is then increased by another four equal increments to the maximum load. The output stresses and strains are recorded for the maximum load. The load then is lowered back to its minimum load, through four increments and its output recorded. The outputs at the minimum and maximum loads are used in updating the constitutive models for the next load step cycle. The load cycle is then increased by a cycle increment, ΔN and the analysis is repeated. The cycle increment ΔN was increased as the analysis progressed until failure of the steel reinforcement was indicated through the Palmgren-Miner rule. If fatigue failure is indicated in the steel reinforcement, as stipulated by the Palmgren-Miner rule, the area of the bar is reduced by one bar (representing fracture of the bar) and the simulation is repeated. This is then repeated, that is one bar area is again removed, until the solution becomes unstable, indicating catastrophic failure under load control.

Table 2- Cycles to failure experimental versus FE result

Series	Beams	Fibre Volumes (%)	Cycles to failure		Variation (%)
			Experiment	FE	
Series 1	1B-1	0.0	211,000	217,301	2.9
	1B-2	0.0	247,000		-13.7
	1C-1	0.4	267,000	325,869	18.1
	1C-2	0.4	369,000		-13.2
	1D-1	0.8	311,000	339,931	8.5
	1D-2	0.8	329,000		3.2
Series 2	2B-1	0.0	351,530	398,831	11.9
	2B-2	0.0	408,118		- 2.3
	2C-1	0.4	522,441	611,376	14.5
	2C-2	0.4	593,959		2.8
	2D-1	0.8	1,053,314	1,030,986	- 2.2
	2D-2	0.8	1,092,535		- 6.0

4. FE RESULTS

The predicted number of cycles to failure is compared with experimental result in Table 2 for each specimen. The predicted failure cycles are very close to experimental result with minimal variations. The stress range, $\Delta\sigma_{N*}$ was chosen to best match the experimental result. The stress range chosen for the analysis is given in Table1.

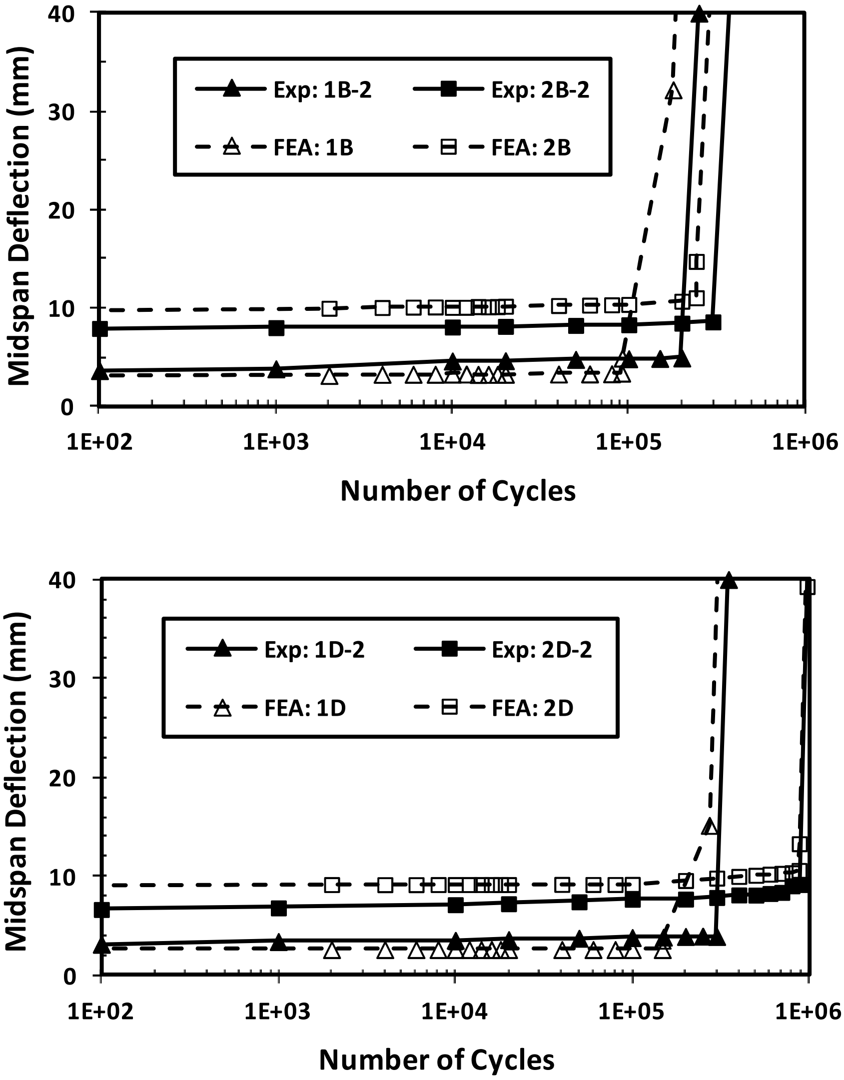


Fig.4- Midspan deflection versus number of cycles at maximum load (a) Beams with 0% fibre (b) Beams with 0.8 % fibres

The midspan deflections at maximum loads for the beams with 0 and 0.8 percent fibres of both series are compared Fig.4. The FE model captures the change in deflection with increasing number of cycles reasonably well; there is an initial period of rapid increase followed by a steady growth period, and then finally a large increase at failure. The FE model shows very good agreement at maximum load, yet underestimates the deflection at minimum load. Loo et al. (2012) also observed this underestimation at minimum load and attributed it to the FE model predicting a greater elastic recovery that what occurred in the experiments. The correlation between increasing fibre content and reduced deflections that was observed in the experiments was also observed in the FE model at the maximum load. The FE results show that the failure is by fracture of the tensile reinforcing steel, as observed in the experiment.

5. CONCLUSIONS

A FE model has been developed for the analysis of SFRC beams subjected to fatigue loading. FEA program RECAP was employed in this study. The beams were modeled using the available material constitutive relationship for the fatigue behaviour of the steel and SFRC. The analysis accounted for cyclic-softening of the concrete and monitored damage of the steel and concrete using the Palmgren-Miner rule. The FE model was tested against the results of two series of test beams. In each series there were beams with 0, 0.4 and 0.8 percent of fibres by volume. Good correlations were observed between the FE analysis results with the tests in terms of displacement at the maximum load and the number of failure cycles. The model overestimated the elastic recovery in obtaining the minimum deflection and further study on this is underway.

REFERENCES

AS 3600 (2009), *Concrete Structures*. Standards Australia.

Comite Euro-International Du Beton (CEB). (1988), "Fatigue of concrete structures, state-of-the-art report." *CEB Bulletin d'Information No. 188*, Lausanne.

Comite Euro-International Du Beton (CEB-FIP). (1993), "Model code 1990." *CEB Bulletin d'Information No. 213*, Paris.

Comite Euro-International Du Beton (CEB). (1996), *RC elements under cyclic loading-state -of–the-art report*, Thomas Telford, London.

Dobromil, P., Jan, C. and Radomir, P. (2010), "Material model for finite element modeling of fatigue crack growth in concrete." *Procedia Engineering*, Vol. 2, 203-212.

Foster S.J. (1992), "An application of the arc length method involving concrete cracking", *International Journal for Numerical Methods in Engineering*, Vol. 33(2), 269-285.

Foster, S.J., Budino, B. and Gilbert, R.I. (1996), "Rotating crack finite element model for reinforced concrete structures." *Computers and Structures*. Vol. 58 (1), 43-50.

Foster, S.J. and Marti, P. (2003), "Cracked membrane model: finite element implementation." *J. Struct. Eng.*, Vol, 129 (9), 1155-1163.

Foster, S.J., Voo, Y.L., and Chong K.T.(2006), "Analysis of steel fiber reinforced concrete beams failing in shear: variable engagement model", in *Finite Element*

Analysis of Reinforced Concrete Structures, Eds. Lowes, L., and Filippou, F., ACI SP-237.

Holmen, J.O., (1982), “Fatigue of concrete by constant and variable amplitude loading.” *Fatigue of Concrete Structures, SP-75*, S.P. Shah ed., American Concrete Institute, Farmington Hills, MI, 71-110.

Kaufmann, W. and Marti, P. (1998), “Structural concrete: cracked membrane model.” *J. Struct. Eng*., Vol. 124 (12), 1467-1475.

Lee, M. K., and Barr, B. I. G., (2004), "An Overview of the fatigue behaviour of plain and fiber reinforced concrete." *Cement & Concrete Composites*, Vol. 26 (4), 299-305.

Loo, K.Y.M., Foster, S.J. and Smith, S.T. (2012), “FE modeling of CFRP-repaired RC beams subjected to fatigue loading.” *J. Compos. Constr., ASCE*, Vol. 16, 572-580.

Mallet, G.P., (1991), *Fatigue of Reinforced Concrete*, HMSO, London.

Miner, M. (1945), “Cumulative damage in fatigue.” *J. Appl. Mech.* Vol. 12(3), A159-A164.

Parvez, A. and Foster, S.J. (2012), “Fatigue behaviour of reinforced concrete beams with addition of steel fibres.” *Proceedings of the 22nd Australasian Conference on the Mechanics of Structures and Materials, ACMSM 22*, Sydney.

Petersson, P.E. (1981), “Crack growth and development of fracture zones in plain concrete and similar materials.” *Report TVBM-1006*, Lund Institute of Technology, Lund.

Schutz, W.(1996), “A History of Fatigue.” *Engineering Fracture Mechanics*, Vol. 54 (2), 263-300.

Tilly, G.P. (1979), “Fatigue of steel reinforcement bars in concrete: a review,” *Fatigue of Engineering Materials and Structures*, Pergamon, Vol. 2, 251-268.

Tilly, G.P. (1985), “Fatigue review: fatigue of land based structures,” *International Journal of Fatigue*, Vol. 7(2), 67-78.

Voo, J.Y.L. and Foster, S.J. (2004), “Tensile fracture of fibre reinforced concrete: variable engagement model”, *Sixth Rilem Symposium on Fibre Reinforced Concrete (FRC)*, Varenna, Italy, 20 22 September,875-884.

Voo, Y.L. and Foster S.J. (2009), *Reactive powder concrete: analysis and design of RPC Girders.* Lambert Academic Publishing.

Yin, W.S., Su, E., Mansur, M., and Hsu, T.C. (1988), “Biaxial tests of plain and fiber concrete,” *ACI Mat.l J.*, Vol. 86 (3), 236-243.