## Fragility Analysis of an in-Service Hysteretic Reinforced Concrete System Subjected to Seismic Actions

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### ABSTRACT

Hysteresis is a phenomenon that characterizes the structural behavior of typical reinforced concrete (RC) structures that undergo load reversals beyond their elastic limits. For an RC structure constructed in an earthquake-prone zone, seismic actions are routinely considered as one of the most commonly encountered situations that may lead to hysteresis. This paper is concerned with the seismic structural safety in relation to a hysteretic RC system. Specifically, the fragility of the RC system at a given level of seismic intensity is quantified. The seismic fragility quantification of this kind is often accompanied by two issues, i.e., incomplete structural appraisal data and prolonged computing time. The former is addressed by using a contemporary statistical technique referred to as the expectation-maximization algorithm, while a surrogate model is rolled out to help resolve the latter. A salient feature of the study presented herein is that the hysteretic system being investigated is presumed to have been in service for a considerable period of time so that its up-to-date, degraded health condition needs to be allowed for.

### 1. INTRODUCTION

Hysteresis is a kind of nonlinear behavior that can often be observed when building materials are loaded beyond their elastic limits. It is particularly prevalent in situations where reinforced concrete (RC) structures are subjected to inelastic load reversals, such as seismic actions, and thus appropriately capturing it has become one of the key performance indicators for relevant analytical or numerical modeling techniques. In this regard, Mostaghel and Byrd (2000) proposed an analytical description of multi-degree-of-freedom dynamic systems with the corresponding hysteretic behavior allowed for. Compared with many other competing analytical or numerical models, the model by Mostaghel and Byrd (2000) stands out thanks to its relative convenience in terms of the

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Fig. 1 A synthetic seismic horizontal ground acceleration record (the upper subfigure) and the resulting horizontal displacement response of the two-story hysteretic RC frame (the lower subfigure)

calibration of the parameters involved and therefore constitutes a promising tool for response characterization of hysteretic RC systems.

In the backdrop of quantification of safety performance of in-service RC systems against seismic actions, this paper reports on a pilot study that in the methodological sense lays some groundwork in the following two respects: efficiency for displacement response simulation and robustness against appraisal data missingness.

# 2. PARAMETER SETUP FOR THE HYSTERETIC RC SYSTEM INVOLVED IN THE PILOT STUDY

A two-story RC shear frame modeled through a two-degree-of-freedom hysteretic system as per Mostaghel and Byrd (2000) is engaged in this pilot study. It is assumed that within the elastic range each story has a normally distributed story stiffness with a mean of  $4 \times 10^4$  kN/m and a coefficient of variation of 0.2. It is further assumed that these two story stiffnesses as a whole follow a bivariate normal distribution with a correlation coefficient of 0.5. A post-yield-to-pre-yield stiffness ratio of 0.15 is introduced to help describe the stiffness reduction associated with the inelasticity. The mass of the first story is taken as a deterministic quantity of 320 t, whereas 300 t is used for the mass of the second story. A Rayleigh damping mechanism is featured in such a way that the modal damping ratios for the pertinent first and second modes are



Fig. 2 Regression analysis in relation to the training, testing, and validating of the surrogate model

0.03 and 0.05, respectively. The seismic-horizontal-ground-acceleration time history used in the study is shown in Fig. 1, which is synthesized based on Clough and Penzien (1993), Soong and Grigoriu (1993), and Kafali and Grigoriu (2007) and has been scaled to have a peak ground acceleration of 2.943 m/s<sup>2</sup> (i.e., 0.3*g* where *g* is the gravitational acceleration). Examples of the resulting horizontal-displacement time histories at the first and roof levels of the RC frame are also plotted as in Fig. 1. The time integration scheme used herein is the fourth-order Runge-Kutta method.



Fig. 3 Regression analysis of the surrogate model output on the output derived from the fourth-order Runge-Kutta method (i.e., the time marching method)

Throughout the fragility analysis performed in this study, the failure of the RC frame is defined as the case that the maximum interstory relative horizontal displacement exceeds 0.15 m, which corresponds to a story drift ratio of 0.03 for a story height of 5 m. It should be emphasized that the limit state with respect to the story drift ratio of an RC frame is highly design-specific (Megally and Ghali 2002; American Concrete Institute (ACI) Committee 318 2014).

#### 3. SURROGATE MODEL AND EXPEDIENT REMEDY

The process of solving the equation of motion of a dynamic system to obtain its displacement, velocity, and acceleration time histories is widely referred to as time integration. On many occasions, time integration is nothing more than invoking an established routine as there exist many canonical time marching methods ranging from straightforward first-order single-point methods (e.g., the explicit Euler method) to more advanced higher-order multi-point methods (e.g., the fourth-order Adams-Bashforth-Moulton method). As for fragility analysis, however, things can sometimes go awry: Rather than being implemented only a few times, time integration often needs to be performed hundreds or even thousands of times in a fragility analysis. As such, the time spent on time integration, which in many other circumstances would be negligible, can really become an encumbrance in a fragility analysis. To address this issue, an attempt was made in this study to construct a surrogate model for a routine time



Fig. 4 Comparison between the time-marching and surrogate-model results

Table 1. An example of a set of the incomplete structural appraisal data for the inservice hysteretic RC system

Realizations of the elastic story stiffness random vector (x10 <sup>4</sup> kN/m)
(3.6009, 3.1777); (3.5557, NA); (2.7020, 2.3300); (4.6302, NA); (4.5615, 3.8353);
(4.3586, 4.1648); (3.3968, 4.3763); (4.4363, 4.6591); (4.2095, 3.8407); (4.9156,
3.2391); (4.1181, 4.8221); (NA, 2.1381); (NA, 4.3149); (3.8054, 3.6278); (3.8863,
3.9124); (3.5433, NA); (5.4288, 4.7475); (4.9363, 4.2828); (3.2399, 2.4820); (4.8827,
NA); (3.7721, NA); (NA, 2.9665); (3.5605, 3.1908); (4.0205, 3.2878); (3.8838, 5.0706);
(NA, NA); (NA, 3.4680); (4.0679, 3.8854); (2.7605, NA); (4.2077, NA).

marching method with the application to the hysteretic RC frame defined in Section 2.

Specifically, with the proven success of the artificial neural network technique in constructing surrogate models for some other engineering applications (e.g., Sreekanth and Datta 2010; Wang et al. 2015), a feedforward backpropagation artificial neural network was created, trained, and validated in the present study by using the Neural Network Toolbox in the commercial software package MATLAB (MathWorks 2013). The network is equipped with two layers with the hidden layer having 20 neurons. The input variables of the network are the realizations of the normally distributed elastic story stiffness random vector, while the maximum interstory relative horizontal displacement of the hysteretic RC frame is the output variable. The training database of the network comprises 2,000 input-output pairs, which were generated by the fourth-



Fig. 5 Comparison of the estimates of the means in the incomplete-data scenario with those in the complete-data scenario

Table 2	Sample	es obtain	ed in the	complete-	and	incomplete	-data	scenarios	for	the
seismic	fragility	of the hy	steretic F	RC system						

Case	Sample
Complete-data scenario	0.0256; 0.0228; 0.0450; 0.0460; 0.0072; 0.0316;
	0.0132; 0.0574; 0.0158; 0.0076; 0.0372; 0.0356.
Incomplete data scenario	0.0290; 0.0132; 0.0358; 0.0598; 0.0478; 0.0474;
incomplete-data scenario	0.0554; 0.0290; 0.0258; 0.0398; 0.0234; 0.0758.

order Runge-Kutta method. The resulting network was validated through the MATLAB toolbox in the first instance, as in Fig. 2. The high coefficients of determination implied by the figure demonstrate the efficacy of the trained network. For the purpose of further validation, corresponding to additional 100 realization units of the elastic story stiffness random vector, two sets of 100 output data points were derived from the fourth-order Runge-Kutta method and the trained network, respectively. These two sets of data are then compared in Figs. 3 and 4. Note that the shaded belt in Fig. 4 marks the zones such that if a data point obtained from the trained network (as indicated by a black solid dot in the figure) falls into them, the relative error of this data point with respect to that yielded by the time marching method is less than 5%. As one can see, Figs. 3 and 4



Fig. 6 Comparison of the estimates of the variances and covariance in the incompletedata scenario with those in the complete-data scenario

appear to further confirm the efficacy of the trained network.

Another issue that should be considered when dealing with in-service civil structures is the possible incompleteness of relevant structural appraisal data (Wang et al. 2015). As an example, Table 1 lists a set of incomplete structural appraisal data simulated for the hysteretic RC frame being investigated, where a missing data point is denoted by an "NA". Instead of chasing up the missing data points, which may not always be viable in reality, this study resorts to the expectation-maximization algorithm (McLachlan and Krishnan 2008; R Core Team 2015; Novo and Schafer 2013) for a remedy. Although the remedy seems to be an expedient only, it does in many cases provide an effective way to handle various missing data situations. This is illustrated by Figs. 5 and 6 through estimating the population distribution parameters pertaining to the elastic story stiffness random vector and by Table 2 in terms of the seismic fragility computation for the hysteretic RC frame. In fact, the two samples shown in Table 2, which were respectively obtained in a complete-data scenario and an incomplete-data scenario with a data missingness probability of 0.3, can be deemed not significantly different from each other based on the result given by a two-sample Kolmogorov-Smirnov test at the commonly used significance level of 0.05.

### 4. CONCLUDING REMARKS

The pilot study presented in this paper explores the feasibility of utilizing appropriately trained artificial neural networks to form a surrogate model to compute the nonlinear response of a representative hysteretic RC frame under seismic actions. The surrogate model thus constructed can be easily deployed to statistical simulation based seismic fragility analysis, so that the computation time needed for the analysis can be shortened phenomenally. The study also investigates the possibility of introducing a mechanism whereby the risk of having access to only incomplete structural appraisal data can be mitigated to some extent when evaluating the seismic fragility of an existing RC structural system.

With the groundwork laid by the pilot study, the ongoing and future research work includes enriching the framework presented here with additional seismic intensity measures and further calibrating the parameters involved in the hysteretic RC system.

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