# Residual stress analysis in composite elastoplastic discs under thermal loading

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# ABSTRACT

Under thermal loading, elastoplastic responses of metallic materials are significantly affected by their temperature-dependent material parameters, such as elastic moduli and yield stress. In this work, we adopt finite element method to study a composite disc being fully constrained on its outer surface under temperature loading and unloading. The composite disc consists of a purely elastic inclusion embedded in elastoplastic material whose properties are temperature dependent. Residual stresses are analyzed, according to the Mises-type yield criterion with the elastic-perfectly plastic model. It is found that a larger inclusion may introduce large amount of residual stress in the matrix.

## 1. INTRODUCTION

Many engineering applications require to use elastoplastic materials, such as metal, under high temperature cycling. Such thermal loading may induce plastic deformation under certain geometric constraints. Considerations of temperature-dependent material properties, such as yield stress, elastic constants, and thermal expansion coefficient, are necessary in order to correctly model the responses of materials (Noda 1991). In addition, shakedown of elastic-perfectly plastic materials has been rigorously analyzed with temperature-dependent elastic moduli (Peigney 2014). Recently, Zarandi et al. (2016) have demonstrated by using the finite element numerical method to study the elastoplastic behavior of composite disc under quasi-static temperature loading.

In this work, we adopt the finite element method to analyze the composite disc under a temperature loading cycle, and study the effects of inclusion size on the magnitude and distribution of residual stress.

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#### 2. NUMERICAL MODELING

As shown in Fig. 1, the composite disc, consisting of an purely elastic inclusion embedded in the elastoplastic matrix with temperature-dependent material properties, same as reported in Zarandi et al. (2016). Only quarter of the disc is analyzed due to the symmetry. The outer boundary of the disc is fully constrained, and the interface between the inclusion and matrix is perfect. The matrix is elastic-perfectly plastic material, and obeys the Mises yield criterion under the plane stress conditions, as follows.

$$\sigma_r^2 + \sigma_\theta^2 - \sigma_r \sigma_\theta = \sigma_0^2 f^2(T).$$
<sup>(1)</sup>

Here the radial and circumferential stress components are denoted by  $\sigma_r$  and  $\sigma_{\theta}$ . At the room temperature  $\sigma_0$  is the yield stress and f(T) is the temperature dependence function for the yield stress. Three different models with inclusion radius a = 0.1, 0.3 and 0.7 m are studied in this work to illustrate the effects of inclusion size on residual stress.



Fig. 1 Finite element mesh for the composite disc with the inclusion radius a = 0.3 m. Axis labels are in units of m.

## 3. RESULTS AND DISCUSSION

Since only rate-independent plasticity is considered, the temperature loading and unloading are applied to the disc in the stepwise manner. Fig. 2 shows the temperature profile used for all of the three models. To avoid sharp corners on the profile, all transition points have been smoothed for at least twice differentiable. The reference temperature is 298 K. The highest temperature, about 395 K, was chosen to cause plastic collapse in the a = 0.7 model.

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Fig. 2 Temperature loading cycle.

The Mises stresses of the a=0.1, 0.3 and 0.7 model are shown in Fig. 3 for two different temperature loading stages. At the temperature loading stage 1, which corresponds to applied temperature being about 320 °C, the deformation of all three models are purely elastic. At the temperature loading stage 2, which corresponds to the highest temperature in the profile, plastic collapse is observed in the a=0.7 model. The a=0.1 and 0.3 model shows small plastic regimes are developed. The size of plastic regime developed depends on the size of inclusion. The larger the inclusion is, the larger the plastic regime would be developed. As for the residual stress, the larger the inclusion is, the larger, in terms of regime size and magnitude, residual stress can be developed. Hence, the a = 0.7 model shows the maximum residual stress at the inclusion-matrix interface about 180 MPa in the Mises sense. The affected regime by the residual stress is the same as the developed plastic regime. Since we assume the inclusion is purely elastic, the stress inside the inclusion is uniform, in consistency with the Eshelby's uniformity theorem.



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Fig. 3 Residual effective stress after a temperature loading cycle for the disc's inclusion radius being (a) a=0.1, (b) a=0.3 and (c) a=0.7 m.

#### 4. CONCLUSIONS

Under a temperature loading cycle, the composite disc, consisting of an elastic inclusion embedded in elastoplastic matrix, may develop different residual stress distributions in accordance to the inclusion radius. With the considerations of temperature dependent material properties, maximum Mises stress is different in the three models. Furthermore, larger inclusions may cause larger residual stress. The maximum residual stress occurs at the interface between the inclusion and matrix.

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