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A new impedance based sensitivity model of piezoelectric resonant cantilever sensor

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ABSTRACT

For piezoelectric cantilever mass sensors, the sensitivity depicting model is of great importance for guiding the sensor design and the corresponding measuring strategy. For example, the existed frequency based sensitivity model evaluated by the frequency shift has inspired a great many structure design methods, and also resulted in very complex frequency sweeping and data acquisition procedures for tracking resonant frequency. The newly proposed simplified method of monitoring impedance change at a fixed frequency from Xu and Mutharasan has been experimentally validated to be effective and superior in sensitivity in hash environments. However, the corresponding sensitivity model has not been established for further enhancing the sensor performances. In this paper, a sensitivity model considering the mechanical-electrical coupling effects is established for evaluating the impedance variation induced by mass-change at a fixed frequency. Importantly, the complex relationship among the exciting frequency, geometrical parameters, and resulted impedance has been theoretically constructed, with which, the structure and control parameters including the fixed exciting frequency and the linear impedance-frequency range can be optimized and adjusted for different applications. A macro-sized cantilever sensor of 26.0 mm long is fabricated, and the measured impedance sensitivity is almost 55.9 times greater than the resonant frequency based sensitivity. Thus, the effectiveness of the impedance based method is adequately verified for the cases where frequency based method fails.

1. INTRODUCTION

Piezoelectric resonant cantilever mass sensor (PRCS) can quantitatively weigh and detect unknown analyte by measuring the dynamic response differences induced by the mass-change. By using piezoelectric material acting both as the actuating and sensing element, such sensors with the elegance of extremely high sensitivity can be conveniently applied to various application fields such as proteomics[1, 2], gas sensing[3], food contamination[4], cancer detecting[5], chemical or fluidic detection[6].

Ordinarily, the analyte weight is always evaluated by measuring the resonance frequency shift induced by the added mass. And then, with the of the piezoelectric cantilever, and the sensitivity can be depicted by the frequency shift per unit mass change ($\Delta f/\Delta m$). Within the frame of such mass sensing mechanism, improving sensitivity means the enhancement of the detection capability for small particles. Hence, a great many sensitivity improving methods have been proposed like the geometrical dimension reduction method[7-9], high-order mode oscillation method[10-12], and cantilever configuration optimization method[13-15]. For example, the significantly improved sensitivity can reach 0.104MHz/zg while the ultra-high resonant frequency is close to 328.5 MHz[16]. Attractive though the high sensitivity is, there exist two complex prerequisite steps of frequency sweeping and data extracting to complete such frequency based measurement, which require high precision frequency meter and fast data extracting and processing instruments. And it is really a time-consuming process to sweep frequency in a relatively wide frequency range. Thus, it appears that the frequency sweeping procedure has become an inherent barrier for portable and convenient usual measurement.

Actually, simple measuring instrument with high sensitivity is of great importance for scientific and industrial applications requiring mass weighing. In order to avoid the frequency sweeping and data acquisition procedure, Xu and Raj Mutharasan[16, 17] firstly proposed the method of measuring the impedance response at **a fixed frequency** near the resonant frequency, and then experimentally validated its effectiveness. Such fixed frequency impedance monitoring approach not only can breakthrough limitation of the frequency-sweep method requiring complex measuring instruments, but also can significantly increase the sensitivity compared to the frequency shift method. Additionally, the fixed frequency method can also reduce the detection time. Nevertheless, it is also pointed out that the experimentally validated method is unsuitable for high Q sensors as the near-linear region in the impedance profile is small. However, the controversial experimental results in reference[23] show a relatively broad linear region over a span of 10 kHz about two times higher than the resonant frequency changes (<5kHz), which is really in contradiction with the conclusion of small detection range. Hence, the key factors influencing the detection range should be carefully analyzed to improve the measuring method. Particularly, a theoretical model is required to explore the mechanical-piezoelectric effects to reveal the relationship among the impedance variation, structure parameters and excitation frequency of such impedance measuring method, which is really useful for optimal design of sensors. Unfortunately, the theoretical sensitivity model has not been found for the impedance approach at a fixed frequency.

Actually, integrating both functions of sensing and actuating, the impedance based mass measurement of the piezoelectric mass sensor is really a multi-physical-field coupling process including the piezoelectric actuated mechanical vibration (inverse piezoelectric effect), the dynamic response to mass-change (dynamic analysis) and the equivalent electric circuit analysis (piezoelectric effect). Due to the complex electromechanical coupling nature of piezoelectric materials, great efforts have focused on the development of the equivalent circuit model. However, to the best knowledge of authors, there has not been an effective piezoelectric-mechanical coupling model for the impedance based mass sensor operating at a fixed frequency, which had been regarded as a simple and convenient mass measuring method. Hence, it is urgent to further explore such mass detection method by using the equivalent circuit theory.

In this paper, to establish the theoretical model for studying the mass weighing approach at a fixed exciting frequency, a mechanical-electrical coupling model is established for calculating the impedance variation induced by mass-change. Through the impedance analysis of the simplified mass weighing approach, the complex relationship among the exciting frequency, geometrical parameters, and dynamic response has been theoretically analyzed, with which, the system parameters such as the fixed exciting frequency and the linear impedance-frequency range can be properly adjusted or designed for different applications. And, with the proposed model, the strategy for selecting the fixed exciting frequency in the vicinity of resonant frequency to achieve maximum sensitivity obtained from Xu and Raj Mutharasan's experiments is theoretically validated. A macro-sized cantilever sensor of 26.0 mm long is fabricated, whose second order resonant frequency is 3114.9Hz about 4.22% deviated from the theoretical value of 3252.0Hz. By exciting the cantilever at the resonance frequency, the measured impedance and sensitivity are $2.49\times10^4\Omega$ and 514.8Ω/mg which are only about 1.92% and 4.4% deviated from the theoretical $2.444\times10^4\Omega$ and 537.5Ω/mg. More importantly, the obtained impedance sensitivity is almost 58.4 times greater than the resonant frequency based sensitivity of 9.2HZ/mg. Considering the manufacturing errors, the theoretical results are nearly consitent with those by experiments, thus, the effectiveness of the proposed equivalent circuit model is adequately verifided, which is really crucial and useful for simplifying the mass weighing procedure and improving sensor performances to desired degree for different applications.

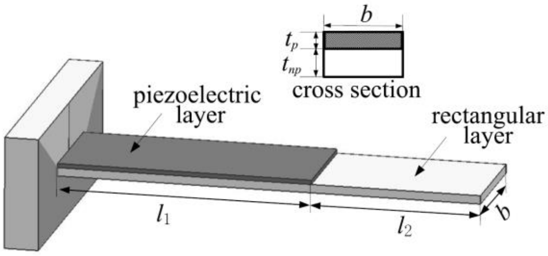


Figure 1 |Structure of piezoelectric cantilever mass sensor

2. Mechanical-electrical coupling model of the cantilever mass sensor

Ordinarily, a piezoelectric layer bonded with an elastic rectangular layer as shown in Fig.1 acts both as a actuating and sensing element in the cantilever mass sensor. When the AC voltage $V(t)=V_0e^{j\omega t}$ is applied on the top and bottom surfaces, the piezoelectric layer will cause the sensor in bending vibration. And meanwhile, the mass change will directly induce changes of the dynamic resonance. In Fig.1, l_1 and l_2 are the length of the piezoelectric layer and the extension respectively, b is the width of the rectangular beam, t_p and t_{np} are the thickness of the piezoelectric layer and the elastic cantilever.

The equation of motion for the lateral vibration of the piezoelectric actuated composite cantilever, including the effect of damping, can be written as

$$m(x)\frac{\partial^2 w(x,t)}{\partial t^2}+c_p\frac{\partial w(x,t)}{\partial t}+\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right)=M(t) \tag{1}$$

where $w(x,t)$ is the transverse displacement, $f(x,t)$ is the applied driving force with frequency ω, $m(x)$ is the mass of unit length, E is the elastic modulus, $I(x)$ is the moment of inertia, c_p is the damping coefficient.

From the piezoelectric actuation principle, the generated equivalent moment *M* acting on the free end of the piezoelectric layer can be obtained. By setting the free end of the piezoelectric layer as the coordinate origin, the equivalent moment *M*(*t*) at the origin can be expressed as follows.

$$M(t) = \frac{bd_{31}E_{np}t_{np}\left(t_p + t_{np}\right)^2}{2s_{11}^E t_p\left(E_{np}t_{np} + E_p t_p\right)\left(E_p t_p + E_p t_{np}\right)} V_0 e^{j\omega t} \tag{2}$$

Where *b* is the width of the piezoelectric layer, d_{31} is the piezoelectric strain coefficient, E_p, E_{np} t_p and t_{np} are the elastic modulus and thickness of the piezoelectric layer and elastic layer, $s^E{}_{11}$ is the elastic compliance coefficient under constant electric field, V_0 the applied voltage amplitude.

By applying orthogonal conditions of the normal modes, the uncoupled ordinary differential equations for the generalized coordinates, $q_n(t)$ can be obtained as

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = Q_n(t) \tag{3}$$

Where ξ_n is the damping ratio of *n*-order mode, ω_n is the n^{th} order natural frequency, *M*(*t*) is the generalized moment.

Then, according to the piezoelectric constitutive equations, the unit surface charge D_3 of the piezoelectric layer can be determined according to the inverse piezoelectric effects.

$$Q = b\int_{-l_1}^{0}\left(\left(\varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E}\right)E_3 + \frac{d_{31}}{s_{11}^E}S_1\right)dx \tag{2}$$

Where S_1 is the surface strain, $s^E{}_{11}$ is the elastic compliance coefficient under constant electric field, $\varepsilon^T{}_{33}$ is the dielectric constant under constant stress, d_{31} is the piezoelectric strain coefficient, and E_3 is the electric field intensity.

Equivalent impedance analyzing for the piezoelectric cantilever mass sensor

According to the BVD equivalent circuit model [18] shown in figure 2, the piezoelectric layer operating at resonance frequency can be simplified as two parallel branches that static capacitance C0 and dynamic parameters including dynamic capacitance (Cm), dynamic inductor (Lm) and dynamic resistance (Rm) respectively. Except for the static capacitance, the dynamic parameters are heavily related to the response of the composite

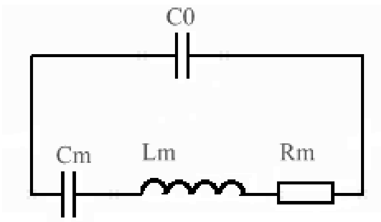


Figure 2 BVD equivalent circuit model from reference [18]

cantilever obtained from the mechanical-electrical coupling analysis.

The static capacitance C0 can be defined according to the capacitance's constitutive equation.

$$C_0 = \frac{l_1 b}{t_p}\left(\varepsilon_{33}^T - \frac{d_{31}^2}{s_{11}^E}\right) \tag{3}$$

According to the circuit theory, the dynamic admittance reaches the maximum value at the resonant frequency, $\omega = \omega_n$.

$$G_{\max} = j\frac{E_{np}t_{np}t_n\varphi_{n1}'(0)H(\omega)\left(t_p + t_{np}\right)^2}{2t_p m_n^e \omega_n\left(E_{np}t_{np} + E_p t_p\right)\left(E_p t_p + E_p t_{np}\right)}\left(\frac{bd_{31}}{s_{11}^E}\right)^2 \int_{-l_1}^{0}\frac{\partial^2\varphi_{n1}(x)}{\partial x^2}dx \tag{4}$$

Also known, the quality factor Q_{nm} can be determined.

$$\begin{cases} H\left(\omega_n\right) = -jQ_{nm} \\ Q_{nm} = \dfrac{1}{2\xi_n} \\ Q_{nm} = \dfrac{1}{R_m}\sqrt{L_m / C_m} \\ \omega_n L_m = 1/(\omega_n C_m) \end{cases} \tag{5}$$

With the dynamic admittance in equation (29), the dynamic capacitance and inductance can be expressed by the known parameters of resonance frequency and quality factor in equation (30).

$$\begin{cases} C_m = \dfrac{1}{\omega_n R_m Q_{nm}} \\ L_m = \dfrac{R_m Q_{nm}}{\omega_n} \end{cases} \tag{6}$$

Then, the Parallel circuit impedance can be determined.

$$Z = R_e + jX_e \tag{7}$$

in which the resistance component and reactance component are

$$\begin{cases} R_e = \dfrac{R_m / (\omega^2 C_0^{\ 2})}{R_m^{\ 2} + \left(\omega L_m - \dfrac{1}{\omega C_m} - \dfrac{1}{\omega C_0}\right)^2} \\ X_e = \dfrac{\dfrac{1}{\omega C_0}\left[R_m^{\ 2} + \left(\omega L_m - \dfrac{1}{\omega C_m}\right)\left(\omega L_m - \dfrac{1}{\omega C_m} - \dfrac{1}{\omega C_0}\right)\right]}{R_m^{\ 2} + \left(\omega L_m - \dfrac{1}{\omega C_m} - \dfrac{1}{\omega C_0}\right)^2} \end{cases} \tag{8}$$

Then, the impedance variation ΔZ induced by the added mass can be determined.

$$\Delta Z = \sqrt{\frac{R_m^{\ 2}}{\alpha} + \frac{\left(R_m^{\ 2} + \left(\omega_n L_m - \dfrac{1}{\omega_n C_m}\right)\left(\omega_n L_m - \dfrac{1}{\omega_n C_0} - \dfrac{1}{\omega_n C_m}\right)\right)^2}{\alpha}} + \sqrt{\frac{\left(R_m^{'\ 2} + \left(\omega_n^{'} L_m^{'} - \dfrac{1}{\omega_n^{'} C_m^{'}}\right)\left(\omega_n^{'} L_m^{'} - \dfrac{1}{\omega_n^{'} C_m^{'}} - \dfrac{1}{\omega_n^{'} C_0}\right)\right)^2}{(\omega_n^{'} C_0 \beta)} + \frac{R_m^{'\ 2}}{\omega_n^{'4} C_0^{\ 4}\beta}} \tag{9}$$

Where

$$\alpha = \omega_n^4 C_0^2 \left(R_m^2 + \left(\omega_n L_m - \frac{1}{\omega_n C_0} - \frac{1}{\omega_n C_m} \right)^2 \right)^2 \tag{10}$$

$$\beta = \left(R_m'^2 + \left(\omega_n' L_m' - \frac{1}{\omega_n' C_m'} - \frac{1}{\omega_n' C_0} \right)^2 \right)^2 \tag{11}$$

Experiments

The structure parameters of the piezoelectric cantilever micro-mass sensor are listed in Table 1. The impedance curves are plotted for the cantilever sensor operating in the second-order mode, as shown in Fig.3.

Tab. 1 Basic dimensions of the mass sensor

Parameters	Rectangular layer	Piezoelectric layer
length/mm	26.0	8.0
width/mm	4.0	4.0
thickness/mm	0.4	0.2
elasticity modulus/GPa	196.5	76.5
density/ kg/m^3	7.8×10^3	7.5×10^3

From figure 3, it can be seen that the second-order mode resonant frequency of the cantilever sensor is 3252.0Hz. And the impedance varies linearly with the exciting frequency in the frequency range from 3206.0 Hz to 3305.0 Hz. So, any frequency in this range can be chosen as the detection frequency for the mass sensing application. For example, at the resonant frequency, curve 'a' and curve 'b' represent the impedances of two cases that loading mass and not loading mass. Then, the impedance variation between the two curves at frequency 3252.0Hz can be used to determine the weight of the added mass.

b

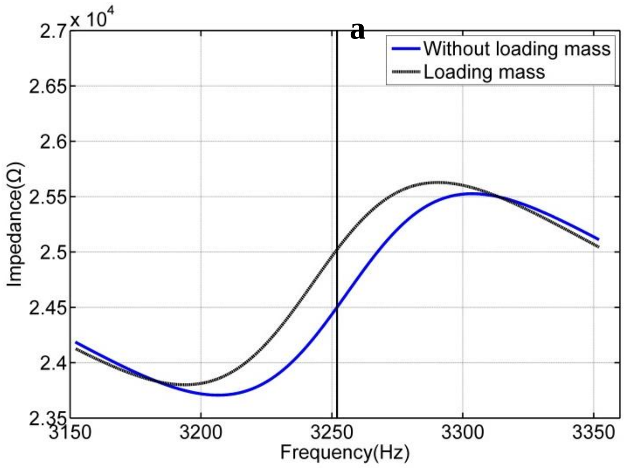
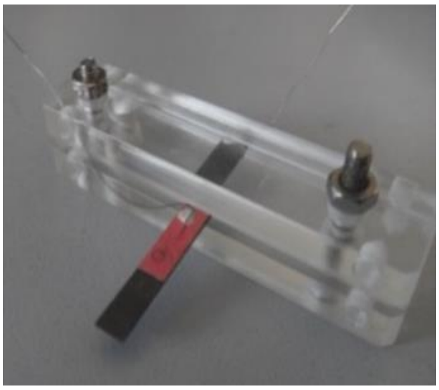
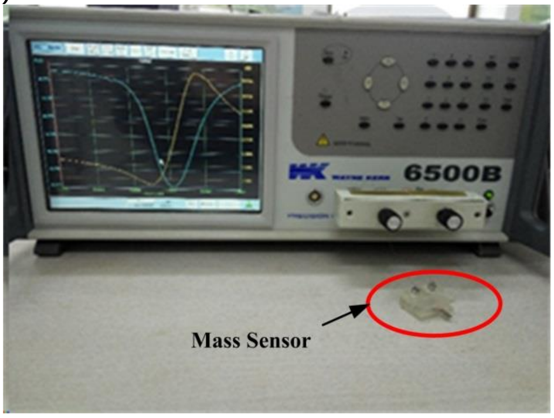


Figure 3| Curve of impedance versus frequency in the vicinity of the second-order mode

In order to validate the effectiveness of the proposed model, a macro-sized cantilever sensor of 4.0mm wide and 26.0mm long is fabricated, whose theoretical second order resonant frequency is 3252.0Hz about 4.22% deviated from the measured value of 3114.9Hz, as shown in figure 4, the sensitivity was measured using a precision impedance analyzer (WK6500B). Due to the impact of conductive adhesive coating and the quality of the connecting wire, the experimental obtained resonant frequency is lower compared to the theoretical results, and thus, resulting in higher impedance.

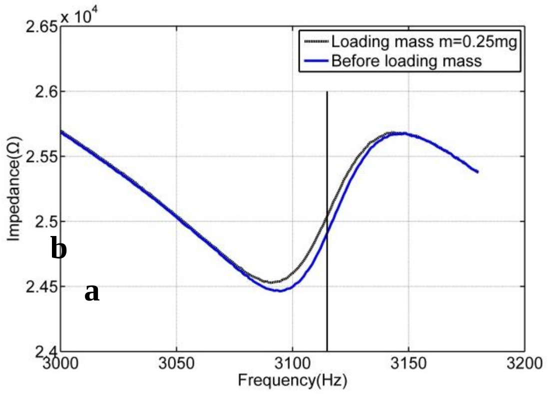


(a)Piezoelectric cantilever micro-mass sensor

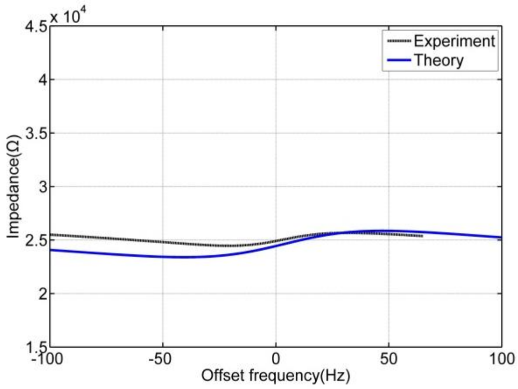


(b)Sensor test system

Figure 5| Sensitivity measurement system



(a) The experimental impedance curve



(b)Impedance curve comparison

Fig.6 Comparison of experimental and theoretical curves

By selecting the resonance frequency of 3114.9 Hz as the fixed exciting frequency, the measured impedance and the sensitivity are $2.49 \times 10^4 \Omega$ and 514.8Ω/mg, which are about 1.92% and 4.4% deviated from the theoretical values of. $2.444 \times 10^4 \Omega$ and 537.5 Ω/mg. Considering influences of the manufacturing and fabricating errors, the deviation between the theoretical and experimental results is in a acceptable level.

Also, the whole linear frequency bandwidth is 22.0Hz. And the theoretical maximum impedance is 2.586e4Ω about 0.7% deviated from the experimental result of $2.568 \times 10^4 \Omega$. Meanwhile, the minimum impedance is $2.342 \times 10^4 \Omega$ about 4.52% deviated from the experimental value of $2.448 \times 10^4 \Omega$. Then, it can be concluded that the proposed theoretical model can precisely predict the detecting frequency range for the cantilever mass sensor.

in the vicinity of the resonant frequency, the bandwidth of the impedance based mass measurement is very narrow, which directly leads to the limitation on the maximum detectable weight of 2.5mg. Generally without the procedure of wide range frequency sweeping, the mass measurement can be quickly completed with a high impedance sensitivity which is almost 58.4 times greater than that from the frequency based method. Hence, such fixed frequency impedance monitoring approach can not only breakthrough limitation of the frequency-sweep method requiring complex measuring instruments, but also can significantly increase the sensitivity and reduce the detecting time, which is really crucial for promoting piezoelectric mass sensors to be applicable in hash detection environments.

Conclusion

A mechanical-electrical model based on the equivalent circuit model is established for calculating the impedance based sensitivity of the resonant mass sensor operating at a fixed frequency. And the complex coupling effects among the dynamic response, structure parameters, excitation frequency and equivalent impedance has been revealed, which is crucial for determining key system parameters e.g. the fixed exciting frequency, the linear impedance-frequency range and piezoelectric parameters. It can also be concluded that the sensitivity varies with the exciting frequency and the optimum exciting point locates at the frequency close to the resonant frequency, which is consistent with the that from Xu and Raj Mutharasan. A macro-sized cantilever sensor of 26.0 mm long is fabricated, whose second order resonant frequency is

3114.9Hz about 4.22% deviated from the theoretical value of 3252.0Hz. By exciting the cantilever at the resonance frequency, the measured impedance and sensitivity are $2.49\times10^4\Omega$ and 514.8Ω/mg which are only about 1.92% and 4.4% deviated from the theoretical $2.444\times10^4\Omega$ and 537.5Ω/mg. More importantly, the obtained impedance sensitivity is almost 58.4 times greater than the resonant frequency based sensitivity of 9.2HZ/mg. Generally speaking, without the procedure of wide range frequency sweeping, the mass measurement can be quickly completed with a high sensitivity which is almost 58.4 times greater than that from the frequency based method. Considering the manufacturing errors, the theoretical results are nearly consitent with those by experiments, thus, the effectiveness of the proposed equivalent circuit model is adequately verifided.

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