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Reliability-based design and topology optimization of structures subject to random excitation

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ABSTRACT

The overarching goal of structural optimization is to find a design solution that provides the best performance while satisfying given design constraints. One of the basic requirements of the structural system design is to withstand stochastic excitations such as seismic and wind loads. The consideration of such loadings directly affects building safety and increases the robustness of building performance. Thus, engineers consider the randomness of the excitement caused by natural disasters in the structure analysis and design process. Due to inherent uncertainties in stochastic loadings to structures, the performance of structural components and system should be assessed in a probabilistic manner. For the reliability assessment of a structure subjected to random excitations, the probability of the occurrence of at least one failure event over a time interval, i.e. the first-passage probability, often needs to be evaluated. In this study, a new method is proposed to integrate probabilistic constraints on the first-passage probability into the structural design and topology optimization. To evaluate the first passage probability effectively during the analysis and optimization, the failure event is described as a series system event consisting of failure events defined at discrete time points, and the system failure probability is obtained with the sequential compounding method. A new sensitivity analysis framework has been developed by integrating the sequential compounding method to facilitate the use of gradient-based optimizers for the proposed method. The proposed optimization framework is successfully applied to the conceptual design of lateral-load resisting systems and space trusses with a desirable level of reliability under earthquake ground motions.

1. INTRODUCTION

Reliable operation of the building system and its sustainability are significant of interest in academic research that directly impacts the industrial development as well

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as the quality of daily living in the social community. It is also intimately linked to safety and survival of people in times of natural disaster where rapid response and recovery along with the reliability of building system and its sustainability play a critical role to reduce the socio-economic losses. Therefore, scientific approach and accountability behind optimizing building structures such as hazard mitigation, predictions of failure, and improvement of structural design processes need to be further developed. The consideration of stochastic excitations such as the wind and seismic loads in structural design directly affects building safety and increases the robustness of building performance. The performance of structural components and system should be assessed in a probabilistic manner due to inherent uncertainties in stochastic loadings to structures. The authors proposed a new method of topology optimization of structures under stochastic excitations (Chun et al. 2013, 2016). In the proposed approach, the failure probability at a given time point, i.e. the instantaneous probability is obtained efficiently by a structural reliability analysis employing a discrete representation method (Der Kiureghian 2000). Although this approach can handle an instantaneous failure probability effectively, it is noted that the performance or reliability of a structure under stochastic excitations often needs to be evaluated over a time interval rather than at a certain time point. Thus, a new system reliability based design and topology optimization method is proposed in this paper to handle constraints on the first-passage probability, i.e. the probability that at least one failure event occurs over a given time duration.

2. RANDOM VIBRATION ANALYSIS USING DISCRETE REPRESENTATION METHOD

2.1. DISCRETE REPRESENTATION OF STOCHASTIC PROCESS

The discrete representation method (Der Kiureghian 2000) discretizes a continuous stochastic process with a finite number of standard normal random variables. A zero-mean Gaussian process *f*(*t*), for instance, can be discretized as

$$f(t) = \sum_{i=1}^{n} v_i s_i(t) = \mathbf{s}(t)^{\mathrm{T}} \mathbf{v} \tag{1}$$

where $\mathbf{s}(t)$ denotes a vector of deterministic basis functions, which is determined by the spectral characteristics of the process (Der Kiureghian 2000), and $\mathbf{v}$ is a vector of n uncorrelated standard normal random variables.

2.2. RESPONSES OF LINEAR SYSTEM UNDER STOCHASTIC EXCITATIONS

For a linear system subjected to a stochastic excitation, the displacement time history *u*(*t*) can be determined by substituting Eq. (1) to Duhamel's integral, i.e.

$$\begin{aligned} u(t) &= \int_0^t f(\tau) h_s(t-\tau) d\tau = \int_0^t \sum_{i=1}^{n} v_i s_i(\tau) h_s(t-\tau) d\tau \\ &= \sum_{i=1}^{n} v_i a_i(t) = \mathbf{a}(t)^{\mathrm{T}} \mathbf{v} \text{ where } a_i(t) = \int_0^t s_i(\tau) h_s(t-\tau) d\tau \end{aligned} \tag{2}$$

where $h_s(t)$ is the unit impulse response function of the degree-of-freedom of interest, and $\mathbf{a}(t)$ denotes a vector determined by solving the convolution integral with $\mathbf{s}(t)$ and $h_s(t)$. The various failure events can then be described in the space of standard normal random variables $\mathbf{v}$. For example, the instantaneous failure event, i.e. the event that the displacement at a certain time $t = t_0$ exceeds a prescribed threshold u_0, is represented by the linear half space $u_0 - u(t_0) = u_0 - \mathbf{a}(t_0)^{\mathrm{T}}\mathbf{v} \leq 0$. From the geometric interpretation, a reliability index can be computed as a closed-form solution, i.e.

$$\beta(u_0, t_0) = u_0 / \|\mathbf{a}(t_0)\| = \hat{\boldsymbol{\alpha}}(t_0)\mathbf{v}^* \tag{3}$$

where $\hat{\boldsymbol{\alpha}}(t_0)$ denotes the negative normalized gradient vector of the limit-state function evaluated at the so-called design point $\mathbf{v}^*$.

3. THE FIRST-PASSAGE PROBABILITY

The first-passage probability, i.e. the probability that a stochastic response exceeds a given threshold at least once for a given duration, is often used to describe the reliability of a system subjected to stochastic excitations (VanMarcke 1975, Fujimura and Der Kiureghian 2007). One of the available approaches to obtain the first-passage probability is defining the problem as a series system problem. i.e.

$$P_{fp} = P(u_0 < \max_{0<t<t_n} u(t)) = P\left(\bigcup_{k=1}^{n} \{u(t_k) > u_0\}\right) \tag{4}$$

This approach requires evaluating the component failure probability at each discrete time point within an interval and performing a system reliability analysis using an efficient, reliable and robust algorithm that can account for statistical dependence between the component events. To handle a large number of component events required by the first-passage probability in Eq. (4), the sequential compounding method (SCM; Kang and Song 2010) is implemented in this research.

4. OPTIMIZATION OF STRUCTURES UNDER FIRST-PASSAGE PROBABILITY

One of the goals in structural optimization is to find the optimal solutions in a given design domain Ω subjected to tractions and displacement boundary conditions while satisfying given design constraints. A formulation of structural optimization under stochastic excitation with first-passage probability constraints can be formulated as

$$\begin{aligned}
&\min_{\mathbf{d}} \;\; f_{obj}(\mathbf{d}) \\
&s.t \;\;\; P(E^i_{sys}) = P\left(\bigcup_{k=1}^{n} E_{f_i}(t_k, \mathbf{d})\right) = P\left(\bigcup_{k=1}^{n} \{g_i(t_k, \mathbf{d}) \leq 0\}\right) \leq P_{f_i}^{\text{target}}, \;\; i = 1, \ldots, n_c \\
&\qquad\;\; \mathbf{d}^{\text{lower}} \leq \mathbf{d} \leq \mathbf{d}^{\text{upper}} \\
&\text{with } \mathbf{M}(\mathbf{d})\ddot{\mathbf{u}}(t, \mathbf{d}) + \mathbf{C}(\mathbf{d})\dot{\mathbf{u}}(t, \mathbf{d}) + \mathbf{K}(\mathbf{d})\mathbf{u}(t, \mathbf{d}) = \mathbf{f}(t, \mathbf{d})
\end{aligned} \tag{5}$$

where $\mathbf{d}$ is a vector of design variables in a design domain Ω, n is the total number of time points during the interesting time interval. $\mathbf{d}^{lower}$ and $\mathbf{d}^{upper}$ are lower and upper bounds of design variables and **M**, **C** and **K** are the mass, damping and stiffness matrices of the design domain, respectively. $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$, **u,** and **f** are the acceleration, velocity, displacement and force vectors at time t, respectively. The reliability index $\beta(t_i)$ of a constraint in Eq. (5) is evaluated at each discrete time point within time duration t_n.

5. SENSITIVITY ANALYSIS

Sensitivity analysis with respect to various design parameters is essential in efficient gradient-based optimization algorithms. In this paper, a sensitivity formulation employing the adjoint method (Choi and Kim 2005) is proposed for linear systems subjected to stochastic excitations modeled by the discrete representation method. In general, the sensitivity of the system failure probability with respect to a parameter θ is obtained by a chain rule, i.e.

$$\frac{\partial P_f(E_{sys})}{\partial\theta} = \sum_{i=1}^{n} \frac{\partial P_f(E_{sys})}{\partial\beta_i(\theta)} \cdot \frac{\partial\beta_i(\theta)}{\partial\theta} \tag{6}$$

Recently, the authors (Chun et al. 2015) proposed a method to compute the derivatives of the system failure probability with respect to the reliability index by use of the SCM. The proposed method enables one to compute sensitivities of a parallel, a series, as well as a general system with respect to reliability indices efficiently. For probabilistic constraints associated with stresses under stochastic excitations aforementioned, the derivative of reliability index in which β_i can be replaced by $\beta(t_i)$ with respect to the parameter d^e can be obtained as follows:

$$\frac{\partial\beta(t_k,\mathbf{d})}{\partial d^e} = -\frac{(L^e\sigma_o / E^e)\left(\sum_{k=1}^{j} b_k(t_j,\mathbf{d}) \cdot \frac{\partial a_k(t_j,\mathbf{d})}{\partial d^e}\right)}{\left(\sum_{k=1}^{j} a_k(t_j,\mathbf{d})^2\right)^{3/2}} \tag{7}$$

where E^e denotes Young's modulus, L^e represents the length of the element e, σ_o is the stress threshold value. To compute the sensitivities, the adjoint method is utilized in the paper. The basic idea of the adjoint method is introducing an adjoint system of equations so that computing implicitly defined terms e.g. $\partial a_k(t_k,\mathbf{d})/\partial d^e$ in sensitivity analysis can be avoided, resulting in reduced computational cost. Detailed discussions about the adjoint system of equations and computing procedure of the partial derivatives can be found in Chun et al. (2016). The final adjoint sensitivity equation yields the followings:

$$\begin{aligned}\frac{\partial P_{fp}(E_{sys})}{\partial d_i} = \sum_{j=1}^{n} \boldsymbol{\lambda}_{n-j+1}^{\mathrm{T}} \Bigg[& \frac{\partial \underline{\underline{\mathbf{A}}}(\mathbf{d})}{\partial d_i} \cdot \mathbf{u}(t_j, \mathbf{d}) - \eta\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_j, \mathbf{d})}{\partial d_i} - (0.5+\gamma-2\eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-1}, \mathbf{d})}{\partial d_i} \\ & -(0.5-\gamma+\eta)\left(\Delta t\right)^2 \frac{\partial \mathbf{f}(t_{j-2}, \mathbf{d})}{\partial d_i} + \frac{\partial \underline{\underline{\mathbf{B}}}(\mathbf{d})}{\partial d_i} \cdot \mathbf{u}(t_{j-1}, \mathbf{d}) + \frac{\partial \underline{\underline{\mathbf{E}}}(\mathbf{d})}{\partial d_i} \cdot \mathbf{u}(t_{j-2}, \mathbf{d}) \Bigg] \\ & + \boldsymbol{\lambda}_n^{\mathrm{T}} \left[\underline{\underline{\mathbf{B}}}(\mathbf{d}) \cdot \frac{\partial \mathbf{u}(0, \mathbf{d})}{\partial d_i} + \underline{\underline{\mathbf{E}}}(\mathbf{d}) \cdot \frac{\partial \mathbf{u}(t_{-1}, \mathbf{d})}{\partial d_i} \right] + \boldsymbol{\lambda}_{n-1}^{\mathrm{T}} \left[\underline{\underline{\mathbf{E}}}(\mathbf{d}) \cdot \frac{\partial \mathbf{u}(0, \mathbf{d})}{\partial d_i} \right]\end{aligned} \quad (8)$$

where **λ** denotes the adjoin vector, T_i is adjoint coefficients, and $\mathbf{y}^{\mathrm{T}}$ is directional cosine vector. For simplicity in the derivation, the following notations are introduced:

$$\begin{aligned}\underline{\underline{\mathbf{A}}}(\mathbf{d}) &= \mathbf{M}(\mathbf{d}) + \gamma\Delta t \cdot \mathbf{C}(\mathbf{d}) + \eta\left(\Delta t\right)^2 \mathbf{K}(\mathbf{d}) \\ \underline{\underline{\mathbf{B}}}(\mathbf{d}) &= -2\mathbf{M}(\mathbf{d}) + (1-2\gamma)\Delta t\mathbf{C}(\mathbf{d}) + (0.5+\gamma-2\eta)\left(\Delta t\right)^2 \mathbf{K}(\mathbf{d}) \\ \underline{\underline{\mathbf{E}}}(\mathbf{d}) &= \mathbf{M}(\mathbf{d}) + (\gamma-1)\Delta t\mathbf{C}(\mathbf{d}) + (0.5-\gamma+\eta)\left(\Delta t\right)^2 \mathbf{K}(\mathbf{d})\end{aligned} \quad (9)$$

6. NUMERICAL APPLICATION

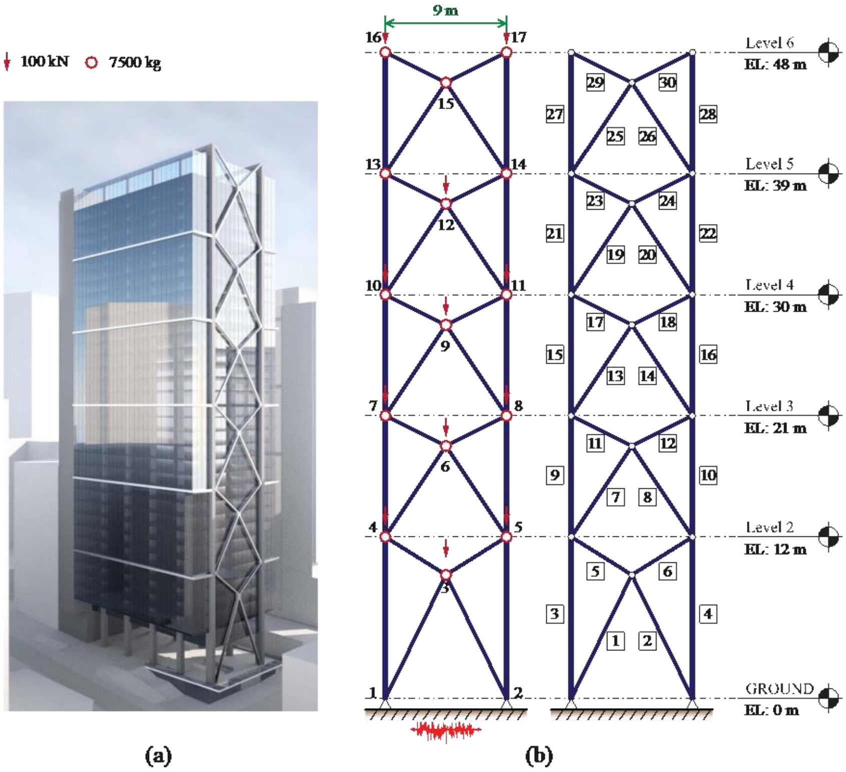

Fig. 1 (a) Rendering of a bracing system (image courtesy of Skidmore, Owings & Merrill, LLP), (b) design geometry and loading conditions

The proposed method is applied to identify optimal member sizes of the lateral bracing system (Fig. 1) subjected to a stochastic earthquake ground motion. The formulation in Eq. (5) is used, while the volume of a design domain is considered as an objective function. Three probabilistic constraints (Case I - compressive stress, Case II - maximum tip drift ratio, and Case III – inter-story drift ratios) associated with the first-passage probability are considered in optimization. The stochastic seismic excitation $f(t)$ is modeled as a filtered white-noise process using the Kanai-Tajimi filter model with the intensity Φ_0 as below (Chun et al. 2016):

$$h_f(t)=\exp(-\zeta_f\omega_f t)\left[\frac{(2\zeta_f^2-1)\omega_f}{\sqrt{1-\zeta_f^2}}\sin(\omega_f\sqrt{1-\zeta_f^2}\cdot t)-2\zeta_f\omega_f\cos(\omega_f\sqrt{1-\zeta_f^2}\cdot t)\right] \quad (10)$$

where ω_f (=5π) and ζ_f=0.4 are filter parameters representing the predominant frequency and the bandwidth of the process, respectively while Φ_0 represents the power spectral density of the underlying white noise process. The force vector in Eq. (5) is described as $\mathbf{f} = -\mathbf{ML}f(t)$ where the vector $\mathbf{L}$ represents directional distribution of masses with unity. The lateral bracing system shown in Fig. 1 is modeled by truss elements. Young's modulus E = 200 GPa and material density ρ = 7,800 kg/m^3 are used as material properties of steel. The damping matrix is constructed using a Rayleigh damping model (Clough and Penzien 1993). Table 1 summarizes the parameter values used for three cases in optimization.

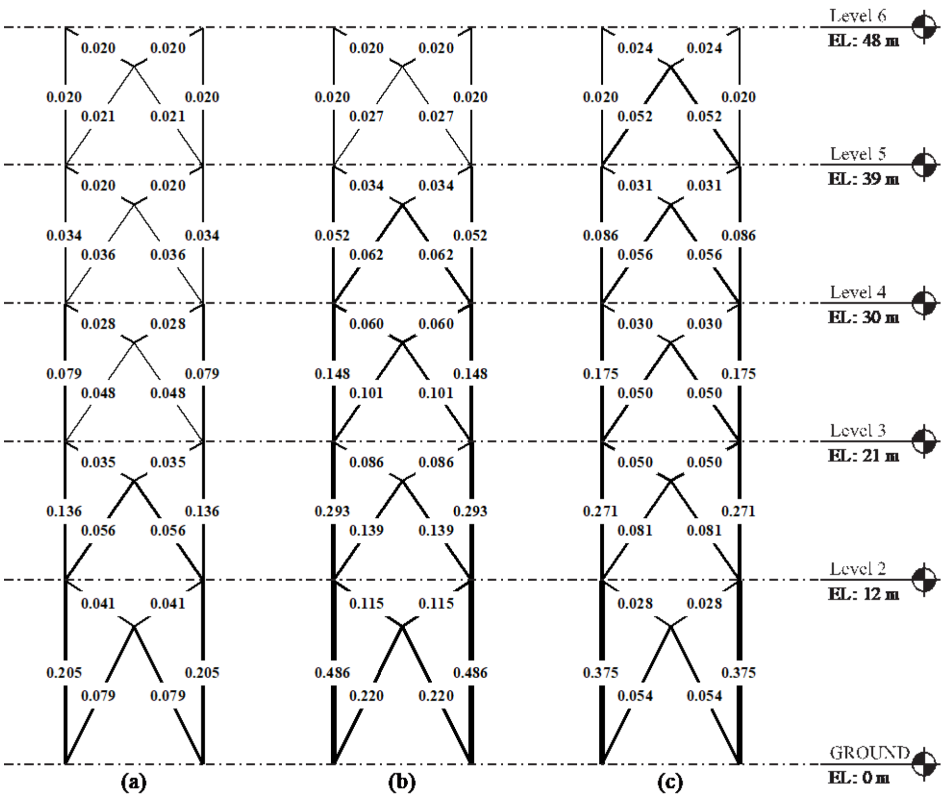

Fig. 2 Optimal truss member sizes: (a) Case I (stress), (b) Case II (maximum tip drift fario), and (c) Case III (story drift ratios)

The proposed method was able to identify the optimal member sizes (Fig. 2) of the lateral bracing system under probabilistic constraints on the first-passage probability. The larger member sizes along the vertical direction for Case II compared to Case III are observable. In Case II, the optimized areas maximize strengthening of the vertical members, whereas the Case III also increases the bracing members as well as the vertical members. It can be because the effects of vertical members to control story drift ratios are significant than X - bracings. Also, it should be noted that the different threshold value, characteristic parameters of the filter, target reliability index will also result in different optimal solutions.

Fig. 3 shows the convergence histories of the objective function and the system failure probability for three cases. The proposed method achieves the system target failure probability effectively. The method quickly identifies an area in the design domain that satisfies the target system reliability, while most of the remaining design iterations identify the minimum volume in the area. Fig. 4 compares dynamic performance of the initial system and optimized system in terms of stress in member #4, a maximum drift ratio and a story drift-ratio. Improved dynamic behaviors, i.e. diminished magnitude of the response to the same input force, are observed in the optimized system.

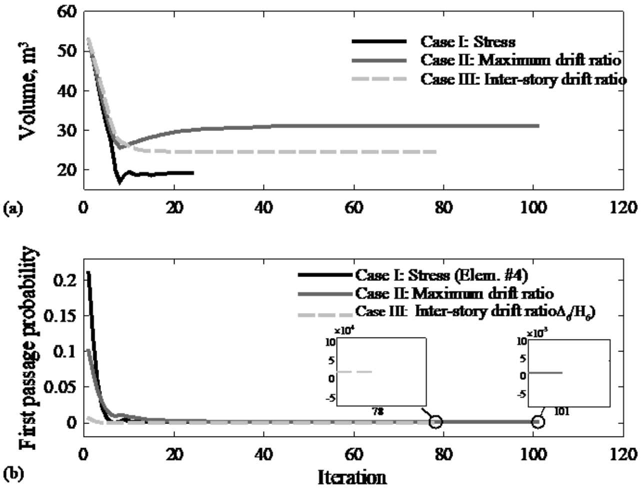

Fig. 3 Convergence histories: (a) volume, and (b) first-passage probability

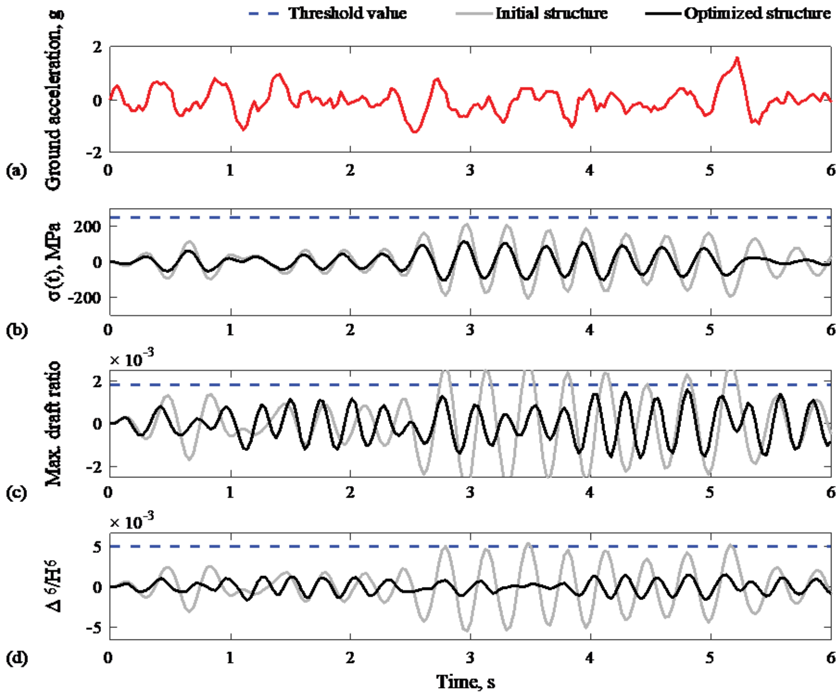

Fig. 4 Dynamic performance: (a) randomly generated excitations, and (b)-(d) corresponding dynamic responses (stress, maximum drift ratio, and inter-story drift ratio, respectively) by initial and optimized systems

Table 1 Parameters used for design domain, probabilistic constraint and ground motion model

Case	Φ_o	Initial member size	Threshold	t_n , sec	β^t_{sys}	P^t_{sys}
I	0.2	0.25 m^2	200 MPa	6.0	3.0	0.0013
II	0.2	0.25 m^2	1/550	6.0	3.0	0.0013
III	0.2	0.25 m^2	1/250	6.0	3.0	0.0013

7. CONCLUSION

In the paper, a new method is developed to incorporate the first-passage probability into structural optimization. Sensitivity calculation of the probabilistic constraint on the first-passage probability is derived to use efficient optimization algorithms. The developed method is successfully applied to the lateral bracing system of structures subjected to the stochastic ground motion to find optimal member sizes. Those can resist future realization of stochastic processes with a desired level of reliability.

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