Linear shell elements for active piezoelectric laminates

Gil Rama*[1], Dragan Z. Marinkovic[1,2a] and Manfred W. Zehn[1a]

[1]*Department of Structural Analysis, Berlin Institute of Technology, Germany*
[2]*Faculty of Mechanical Engineering, University of Nis*

(Received keep as blank , Revised keep as blank , Accepted keep as blank)9pt

Abstract. Piezoelectric composite laminates are a powerful material system that offers vast options to improve structural behavior. Successful design of piezoelectric adaptive structures and testing of control laws call for highly accurate, reliable and numerically efficient numerical tools. This paper puts focus onto linear and geometrically nonlinear static and dynamic analysis of smart structures made of such a material system. For this purpose, highly efficient linear 3-node and 4-node finite shell elements are proposed. Both elements employ the Mindlin-Reissner kinematics. The shear locking effect is treated by the discrete shear gap (DSG) technique with the 3-node element and by the assumed natural strain (ANS) approach with the 4-node element. Geometrically nonlinear effects are considered using the co-rotational approach. Static and dynamic examples involving actuator and sensor function of piezoelectric layers are considered.

Keywords: active structure; linear shell element; piezoelectricity; co-rotational FEM; geometric nonlinearity

1. Introduction

More than two decades ago the idea of adaptive/smart structures has seen the light of day. It denoted the conceptual change of structures from passive, deformable systems to active systems capable of sensing changes in their condition and performing adequate actions to resist undesired changes (Gabbert and Tzou 2000). The idea has opened up vast possibilities to improve structural behavior and features such as vibration suppression (Li and Yao 2016, Oveisi and Nestorovic 2016), noise attenuation (Aridogan and Basdogan 2015), shape control (Zhang *et al*. 2016, Zhang *et al*. 2017), energy harvesting (Biswal *et al*. 2005), structural health monitoring (Masmoudi *et al*. 2015), thus offering improved safety, robustness and comfort.

Shell structures with piezoelectric active elements, as a distinctive group of adaptive structures, have drawn a great deal of attention from the research community. This may be attributed to the fact that the majority of engineering structures are thin-walled structures. Additionally, by adding/embedding active elements in the form of thin piezoelectric patches, the thin-walled structures can be converted into adaptive systems in a relatively simple manner. Piezoelectric material based active elements are a common choice for this purpose, as they operate in the required frequency range and offer adequate force, electric voltage and stroke ranges for this type of structures. The piezoelectric patches are used as both actuators (reverse piezoelectric effect) and sensors (direct piezoelectric effect).

Design of such structures calls for accurate, reliable and efficient numerical tools. This enables optimization in the early design stages related to the structure's geometry, size, number and position of active elements as well as different parameters of the control algorithms. The finite element method (FEM) is typically addressed as the most powerful tool in the field of structural analysis. A number of researchers dedicated their work to the development of various finite elements for modeling and simulation of piezoelectric active structures.

Three-dimensional solid elements do not represent the first choice when global structural behavior of thin-walled structures is aimed at. However, they offer insight into some local effects not covered by typical shell elements and were therefore addressed in the work of some researchers. They require additional techniques to improve their performance when used for modeling shell type of structures. Lee *et al*. (2004) developed an 18-node solid element with the assumed strain technique. Braess and Kaltenbacher (2008) used balanced reduced integration in the thickness direction, applied only to a portion of the shear term, to formulate a quadratic hexahedral piezoelectric element. Willberg and Gabbert (2012) based a 3D piezoelectric finite element for smart structures on the isogeometric approach.

A number of researchers aimed at formulations that are essentially two-dimensional but offer accuracy and fidelity close to three-dimensional formulations. Those efforts resulted in layerwise theories. The Carrera Unified Formulation (CUF) (Carrera 2003) was used in a number of element formulations (Cinefra *et al*. 2015a, b). The formulation was also used by Valvano and

∗Corresponding author,
E-mail: gil.rama@tu-berlin.de
[a]Ph.D.

*Note: Copied from the manuscript submitted to
"Smart Structures and Systems, An International Journal" for the purpose of presentation
at ASEM17/ANBRE17.

Carrera (2017) to develop variable kinematic shell elements, in which both the equivalent single layer approach and the layerwise approach are used together to combine their advantages in the case of purely mechanical field. This formulation was extended by Carrera and Valvano (2017) to coupled electro-mechanical problems.

In most developments of piezoelectric shell elements, equivalent single layer based theories were addressed as they offer a very good balance between the accuracy and numerical effort when the global structural behavior is aimed at. The body of literature on the topic is prohibitively large for an exhaustive overview. Both the classical laminate theory (Kirchhoff-Love kinematical assumptions) and the First Order Shear Deformation theory (Mindlin-Reissner kinematical assumptions) were used in the developments, the latter being more often addressed. The developed elements include both numerically highly efficient linear shell elements (To and Liu 2003, Zemčík *et al*. 2007) and curved quadratic shell elements (Gabbert *et al*. 2002, Marinkovic *et al*. 2006) with various techniques implemented to alleviate the locking phenomena. Although most of the developments are for linear analysis, some of the researchers also tackled the problems of geometrically nonlinear analysis in their work (Simoes Moita *et al*. 2002, Rabinovitch 2005, Lentzen *et al*. 2007, Marinkovic *et al*. 2008, Rama 2017). Some of the developed elements were also implemented in commercial software packages by means of user subroutines (Nestorovic *et al*. 2013, Nestorovic *et al*. 2014), while a number of studies were aimed at different aspects in modeling piezoelectric coupled-field effects (Marinkovic *et al*. 2009, Piefort 2002, Marinkovic and Marinkovic 2012, Zhang 2014).

In this paper, two numerically highly efficient, linear shell elements are presented for modeling piezoelectric shell structures with piezopatches polarized in the thickness direction. The formulation of the triangular and quadrilateral elements is extended to geometrically nonlinear analysis. For this purpose a co-rotational approach (Felippa and Haugen 2005, Nguyen *et al.* 2016) is used.

2. Geometry and mechanical field of the elements

For the sake of brevity, in further text the linear triangular and quadrilateral shell elements will be referred to as SH3 and SH4ANS, respectively.

For treatment of the transverse shear locking effect, the SH3 element uses the discrete shear gap technique as proposed by Bletzinger *et al*. (2000) while the SH4ANS element relies on the assumed natural strain (ANS) approach. In addition, to improve the accuracy and stability of the SH3 element, the cell strain smoothing technique proposed by Nguyen-Thoi *et al*. (2013) is implemented.

Beside the global coordinate system, (x, y, z), the formulation of both elements requires also a local coordinate system, (x′, y′, z′), Fig. 1. It is defined so that the local x′-axis is oriented from element node 1 towards node 2, while the z′-axis is perpendicular to the plane defined by the x′-axis and a vector orientated from node 1 to node 3. The local y′-axis is then easily obtained by the cross-product.

The shell geometry with respect to the local coordinate system is practically regenerated from its mid-surface:

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \sum_{i=1}^{n} N_i \begin{Bmatrix} x'_i \\ y'_i \\ 0 \end{Bmatrix} + \sum_{i=1}^{n} \frac{h}{2} N_i t\{ e_{z'} \} \tag{1}$$

where h is the shell thickness, N_i are the shape functions, n indicates the number of nodes of the element (for SH3 n = 3; for SH4ANS n = 4), $\{e_{z'}\}$ is the unit vector of the z′-axis,

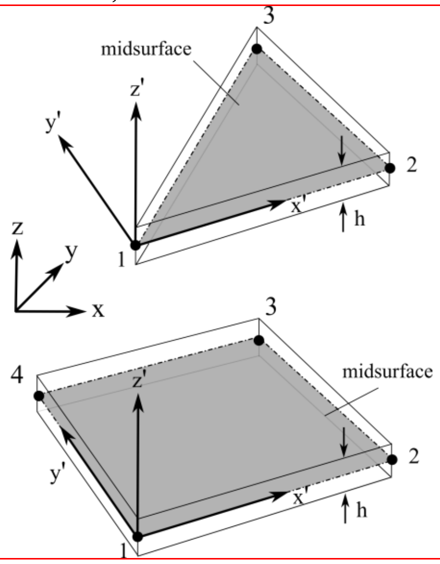

Fig. 1 Geometry and the local coordinate systems of the 3-node and 4-node shell elements