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Dynamic characteristics of double-block ballastless track subjected to a moving harmonic load

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ABSTRACT

The dynamic characteristics and resonance occurrence conditions of the double-block ballastless track under a moving harmonic load are investigated in this study. The dynamic response of the ballastless track under a moving harmonic load are derived using the two-dimensional Fourier transform method. The influences of physical parameters of the under-rail structure on the critical velocities are semi-analytically studied. The results show that when the mass of the concrete block increases, the critical velocity will decrease.

1. INTRODUCTION

The dynamic characteristics of the double-block ballastless track are one of the recent research focuses. Chen (2007) adopted the modal analysis method to analyze the effect of the stiffness of the railpad and block bearing on the vibration of the track in the frequency domain. Xiang (2008) proposed a finite element model for the vertical vibration analysis of this kind of track and derive the total potential energy of vertical vibration of it. Based on this model, He (2010) analyzed the effect of the stiffness of the railpad and block bearing on the deformation of the rail and gave a reasonable range of the stiffness of the track system. Chen (2011) established the vehicle-track coupled dynamics model in which the rail is modelled as a Timoshenko beam based on this kind of track, and adopted a new explicit numerical integration method to solve the vibration equation, and the effects of track irregularities and high frequency loads on track vibration are studied. Hussein (2006) derived the analytical expression of the dynamic response of the floating-slab track with continuous slabs under a harmonic moving load using Fourier transform and residue theorem, Wu (2008) and Yuan (2011) adopted a similar method to analyze the dispersion characteristics of the floating-slab track with short slabs and the dynamic response of this kind of track under a harmonic moving

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load. However, the precious studies did not give the analytical form of the relevant characteristic roots in the dynamic response expressions.

In this paper, the Fourier transform are used to derive the dynamic response of the double-block ballastless track under a moving harmonic load. Based on the analytical model, the influences of the parameters of the track system on the dispersion characteristics and the critical velocity are studied.

2. MODELLING OF THE DOUBLE-BLOCK BALLASTLESS TRACK

The model of the double-block ballastless track (Zhai 2015) is shown in Fig.1. The rail is modeled as an infinite Euler beam (with mass m_1 per unit length and bending stiffness *EI*), the physical properties of the railpads (with stiffness k_1 per unit length and damping factor c_1 per unit length) and block bearings (with stiffness k_2 per unit length and damping factor c_2 per unit length) are characterized by continuously distributed springs and dampers. Only the inertial effect of the concrete blocks (with mass m_2 per unit length) is considered as its length is very short. In this paper, the train loads are simplified to a moving harmonic load with frequency ω_0 and velocity v which can be written as $F(t) = e^{i\omega_0 t}\delta(x - vt)$.

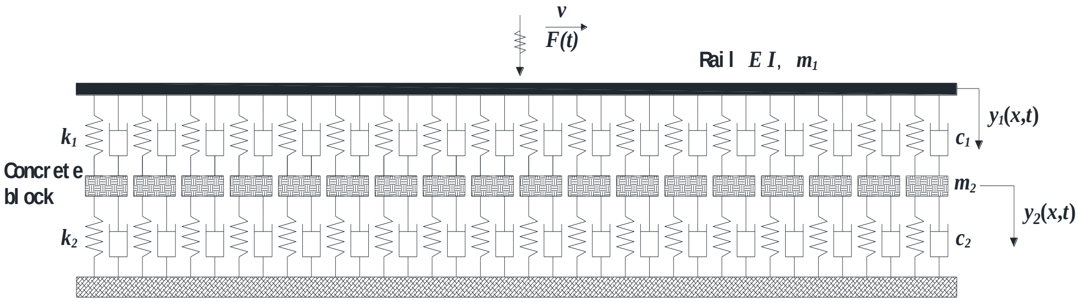

Fig. 1 Modelling of the double-block ballastless track under a moving harmonic load

The vibration equations of the track system can be written as

$$EI\frac{\partial^4 y_1}{\partial x^4} + m_1\frac{\partial^2 y_1}{\partial t^2} + k_1(y_1 - y_2) + c_1\left(\frac{\partial y_1}{\partial t} - \frac{\partial y_2}{\partial t}\right) = e^{i\omega_0 t}\delta(x - vt) \quad (1)$$

$$m_2\frac{\partial^2 y_2}{\partial t^2} + k_2 y_2 - k_1(y_1 - y_2) + c_2\frac{\partial y_2}{\partial t} - c_1\left(\frac{\partial y_1}{\partial t} - \frac{\partial y_2}{\partial t}\right) = 0 \quad (2)$$

Eq. (1) and (2) are transformed to the wavenumber-frequency domain (ξ, ω) using the two-dimensional Fourier transform:

$$(EI\xi^4 - m_1\omega^2 + k_1 + ic_1\omega)Y_1 - (k_1 + ic_1\omega)Y_2 = 2\pi\delta(\omega + \xi v - \omega_0) \quad (3)$$

$$[-m_2\omega^2 + k_1 + k_2 + i\omega(c_1 + c_2)]Y_2 - (k_1 + ic_1\omega)Y_1 = 0 \quad (4)$$

Then the dynamic response of the rail and concrete block are obtained in the wavenumber-frequency domain:

$$Y_1(\xi,\omega) = \frac{2\pi\delta(\omega+\xi v-\omega_0)f_2(\xi,\omega)}{f_1(\xi,\omega)} \quad (5)$$

$$Y_2(\xi,\omega) = \frac{2\pi\delta(\omega+\xi v-\omega_0)f_3(\xi,\omega)}{f_1(\xi,\omega)} \quad (6)$$

where

$$f_1(\xi,\omega) = \begin{vmatrix} EI\xi^4 - m_1\omega^2 + k_1 + ic_1\omega & -(k_1 + ic_1\omega) \\ -(k_1 + ic_1\omega) & -m_2\omega^2 + k_1 + k_2 + i\omega(c_1 + c_2) \end{vmatrix} \quad (7)$$

$$f_2(\xi,\omega) = -m_2\omega^2 + k_1 + k_2 + i\omega(c_1 + c_2) \quad (8)$$

$$f_3(\xi,\omega) = k_1 + ic_1\omega \quad (9)$$

3. DISPERSION CHARACTERISTICS

The dispersion characteristics of the track system is checked firstly. In this study, the double-block ballastless track system described in Table 1 (Cui 2000) is investigated.

Table 1. Physical parameters of the track system

Rail	Concrete block
EI=6.1 MN • m^2	m_2=3280 kg/m
m_1=60.34 kg/m	k_2=7.31 MN/m^2
k_1=100 MN/m^2	c_2=35000 Ns/m^2
c_1=30000 Ns/m^2	

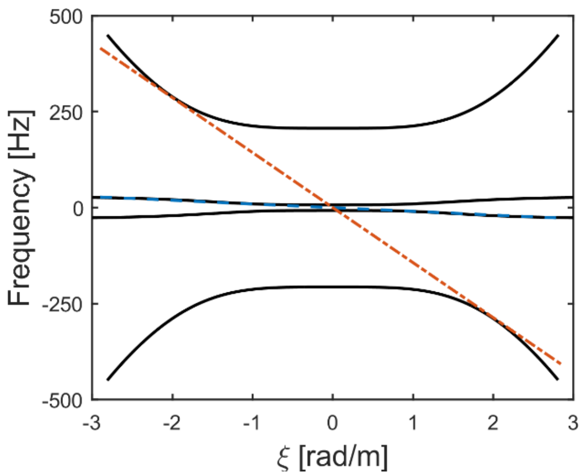

Fig. 2 Dispersion curves (——) of the double-block ballastless track and the load velocity lines for two velocities 60m/s (-----) and 900m/s (-·-·-)

Let $f_1(\xi,\omega)=0$ (c_1=c_2=0 Ns/m^2), the dispersion equation is expressed as

$$(EI\xi^4 - m_1\omega^2 + k_1)(-m_2\omega^2 + k_1 + k_2) - k_1^2 = 0 \tag{10}$$

As shown in fig. 2, the dispersion curves are symmetric about the wavenumber and frequency axes, both dispersion curves in the positive frequency range have a lowest point in which the frequency are called cut-off frequency. For the studied track system, two cut-off frequencies $\omega_{\text{cut-off},1} = 7.4\text{Hz}$, $\omega_{\text{cut-off},2} = 206.73\text{Hz}$ can be obtained. The load velocity line is plotted in the dispersion curves. When the load velocity line is tangent to the dispersion curves, the corresponding velocity is the critical velocity. Fig. 2 shows two tangent cases when the load frequency equals to 0 Hz.

4. CRITICAL VELOCITIES

When the load moving velocity reaches the critical velocity, the amplitude of the track vibration will increase greatly. The derivative of frequency with respect to wavenumber can be got from Eq. 10:

$$\frac{d\omega}{d\xi} = \frac{4EI\xi^3}{2m_1\omega + 2m_2\omega k_1^2(-m_2\omega^2 + k_1 + k_2)^{-2}} \tag{11}$$

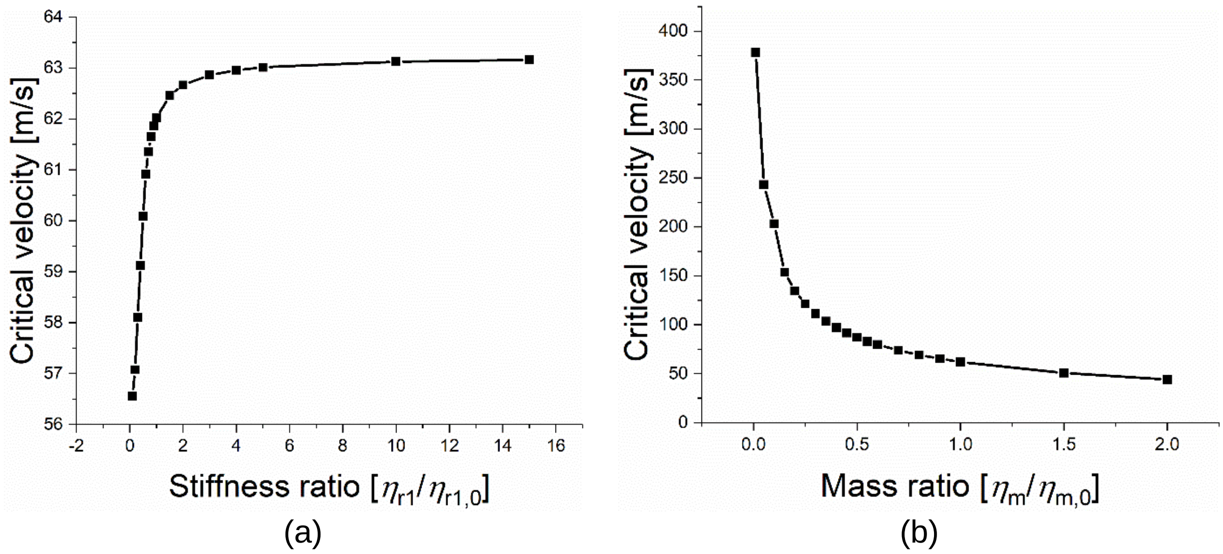

Fig. 3 The variation of critical velocity of the track under a moving concentrated load ($\omega_0 = 0\text{Hz}$) along with the variation of k_1 (a), m_1 (b).

Let $(\xi_{\text{cr}}, \omega_{\text{cr}})$ be the coordinate at the tangent point of the load velocity line ($\omega = \omega_0 - \xi v$) and the dispersion curves, then it must satisfy the conditions described by the following equations:

$$f_1(\xi_{\mathrm{cr}}, \omega_{\mathrm{cr}})\big|_{c_1=c_2=0(\mathrm{Ns/m^2})} = 0, \quad \frac{d\omega}{d\xi}\bigg|_{\xi=\xi_{\mathrm{cr}}} \xi_{\mathrm{cr}} + \omega_0 - \omega_{\mathrm{cr}} = 0 \tag{12a,b}$$

The coordinate of the tangent point can be found from Eq. 12, then the critical velocity can be calculated:

$$v_{\mathrm{cr}} = -\frac{d\omega}{d\xi}\bigg|_{\xi=\xi_{\mathrm{cr}}} \tag{13}$$

Fig. 3 presents the variation of the computed critical velocity of the track under a moving concentrated load ($\omega_0 = 0\mathrm{Hz}$) along with the variation of k_1 and m_1. As shown, as the stiffness of the railpad increases, the critical velocity increases monotonously when k_1 is greater then $0.56k_{1,0}$. The critical velocity will decrease if the mass of the concrete block increases.

5. CONCLUSIONS

In this paper, the influences of the stiffness of the railpad and block bearing and the mass of the concrete block on the dispersion curves and critical velocity are studied. The results show that as the stiffness of the railpad increases, the critical velocity to the resonance occurrence will generally increases. For the mass of the concrete block, as its increase, the critical velocity will decrease.

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