The 2016 World Congress on
The 2016 Structures Congress (Structures16)
Jeju Island, Korea, August 28-September 1, 2016

Modified UKF method study for parameters identification of hysteresis model

*Luyu Li[1)] and Han Qin[2)]

[1), 2)] *School of Civil Engineering, DUT, Dalian, China*

ABSTRACT

This article focuses on adopting the UKF state observer to hysteresis model parameter identification. Scaled UKF are reviewed, after that a modified Unscented Kalman filter (UKF) parameter identification method is proposed. In this method, the one point a step observation often used in UKF identification is replaced with several time points, which are chose randomly in 'future' observed values. The random sample largely decrease the calculation effort of the inverse of the observation covariance matrix, while obtain a more easy converge and robust estimation. At the end of this article, numerical simulation is presented by identify a complete brief Bouc-wen model parameters to show the effectiveness of this method.

1. Introduction

Since Kalman filter first applied by Stanley F. Schmidt in 1960s[1], its application has been extended to various fields, such as global navigation[2], target tracking[3], chemical process control[4], etc. It has been improved for different purposes, and adapted for fit different situations. For state estimate of nonlinear system, there are extended Kalman filter [5], unscented Kalman filter [6], cubature Kalman filter [7], Particle filter [5], etc.

With its feasibility to more complicated nonlinearity relative to extended Kalman filter, lower cost of computation effort relative to particle filter, applicability to more probability distribution relative to cubature Kalman filter, unscented Kalman filter have witness plenty of applications and modifications recently. One of its important applications is parameter identification.

In terms of parameter identification, mainly two forms are frequently used.

One is dual identification.[8] This form identify states of dynamic system and parameters separately with two Kalman class filters interactive with each other, the first Kalman filter regards parameters as known, while the second one views parameters as states and the dynamic states of system as known.[9, 10]

The other is joint identification.[8] This form retains the structure of UKF state observer, parameters are consider as state variables together with the dynamic states of

[1)] Professor
[2)] Graduate Student

system.[11-13]

In both of these two forms, states are calculated based on the last step estimates. As the states and parameters being unknown, only the structure of the system and the observations obtained in current step is available. Uniqueness of states and parameters cannot be ensured in the vicinity of current state, even though the generated observation estimation comply well with the observation. It often course fake identified parameter or instability of the Kalman filter. To fix this problem, Rodrigo Astorza left only parameters as the states of UKF. He acquired the current states of system by integrating from the zero time point with current estimated parameters. [14] As time series become longer, each sigma point require more compute time. Moreover, there is no precise system in nature, each system operate with phase noise, alone with accumulation of inaccurate time counting in sampling of signal acquisition system, error will increase.

In this article, the form of joint identification was used with single step observation replaced by random samples of a section of observed ‘future’ observations. This method increased the uniqueness of the parameters and decreased the calculation effort simultaneously. The second section consists a review of the scaled UKF method and a introduction of the improved method. In the third section a numerical simulation is carried out to show effectiveness of this improved method, the last section will summarize the merits and defects discovered in the implement of this method.

2. Classic UKF and the improved method

2.1 Classic UKF

UKF relies on unscented transform to propagate the mean and covariance of a random variable $\boldsymbol{x}$ through a nonlinear transform $\boldsymbol{y} = \boldsymbol{g(x)}$. In this article, we present an improved version of UKF, called scaled UKF. It was first studied by Simon J. Julie. [15] Simon J. Julie formed 2L+1 points $\boldsymbol{X_i}$, around the estimated mean point $\bar{\boldsymbol{x}}$, with corresponding weights ω_i. These points are called sigma points. With these sigma points we can estimate the transformed mean $\bar{\boldsymbol{y}}$ and covariance $\boldsymbol{P_y}$. The sigma points are formed as follows:

$$\boldsymbol{X_0} = \bar{\boldsymbol{x}}$$

$$\boldsymbol{X_i} = \begin{cases} \bar{\boldsymbol{x}} + \left\{\sqrt{L+\lambda \boldsymbol{P_x}}\right\}_i & i = 1,\cdots,L \\ \bar{\boldsymbol{x}} - \left\{\sqrt{L+\lambda \boldsymbol{P_x}}\right\}_{i-L} & i = L+1,\cdots,2L \end{cases}$$

$$\omega_0^{(m)} = \frac{\lambda}{L+\lambda}$$

$$\omega_0^{(c)} = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta)$$

$$\omega_i^{(m)} = \omega_i^{(c)} = \frac{1}{2(L+\lambda)}$$

$$\lambda = \alpha^2(L+\kappa) - L$$

$\alpha \in (0,1]$ is a scaling parameter, determines the spread of the 2L sigma points $\boldsymbol{X}_i$

around the center point $\boldsymbol{X_0}$, in this research, we set it as 0.01. κ is a secondary scaling parameter which is usually set to 0, and β is used to incorporate prior knowledge of the distribution of $\boldsymbol{x}$ (for Gaussian distributions, $\beta = 2$ is optimal) . $\sqrt{\square}$ calculates the square root of matrix, which is defined as $P = \sqrt{P}^T \square \sqrt{P}$.[16] $\{\square\}_i$ returns the i th column of a matrix. $\square^{(m)}$, $\square^{(c)}$ represent the weight coefficient when estimating mean and covariance.

We get 2L points $\boldsymbol{Y_i} = g(\boldsymbol{X_i})$, after the sigma points passed through the nonlinear transform. Then the mean and covariance after nonlinear transform can be estimated as

$$\bar{\boldsymbol{y}} \approx \sum_{i=0}^{2L} \omega_i^{(m)} \mathbf{Y}_i$$

$$\boldsymbol{P}_y \approx \sum_{i=0}^{2L} \omega_i^{(c)} (\mathbf{Y}_i - \bar{\mathbf{y}})(\mathbf{Y}_i - \bar{\mathbf{y}})^T$$

$\square^T$ denotes the transposition of a matrix. For non-Gaussian variables, the accuracy of third and higher order moments determined by the choice of α and β. Details of the derivation and proof can be found in Simon J. Julie's article. [15]

After review of the unscented transform, let us consider the following single-degree-of-freedom (SDOF) general nonlinear continuous dynamic system in structure dynamics. This model is excited by ground acceleration.

$$m\ddot{x} + c_l \dot{x} + f(x) = -m\ddot{x}_g$$

Divide m in both sides of this equation, we can get

$$\ddot{x} + c\dot{x} + g(x) = -\ddot{x}_g$$

where

$$c = c_l / m$$

$$g(x) = f(x) / m$$

In this equation, x is the displacement relative to the ground, $\dot{\square}$ denotes derivative with respect to time, $\ddot{x}_g$ is the ground acceleration, m is the mass of the SDOF system, c_l is the equivalent damping coefficient, $f(x)$ is the nonlinear stiffness, which can either be elastic or hysteresis or both.

In KF class identification methods, dynamic equations are written in state space form. For joint identification problem (joint identification of states and parameters), the state space equation can be written as follows

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dot{\boldsymbol{\theta}} \end{pmatrix} = \begin{pmatrix} \dot{x} \\ -\ddot{x}_g - c\dot{x} - g(x) \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{w}_s \\ \mathbf{w}_{\boldsymbol{\theta}} \end{pmatrix}$$

Where $\boldsymbol{\theta}$ stands for parameters of the system, $\boldsymbol{w}_s$ represents system noise, which can arise from inaccurate modeling, or noise induced by sensors when measuring ground excitation, etc. $\boldsymbol{\omega_\theta}$ denotes the parameter uncertainty vector, which can also be used for identification strategy, called fake noise.[16]

In this article, we chose displacement x as the observed variable. Thus, for one point observation, the observation function is $\boldsymbol{y} = [1\ 0 \cdots 0]\,\boldsymbol{x} + \boldsymbol{v}$. $\boldsymbol{v}$ indicates the additive observation noise. $\boldsymbol{x}$ represents the augmented vector consist states and parameters. Obviously, the nonlinear transform appear only in the state update process.

The state space representation can be abbreviate as follows.

$$\boldsymbol{x}_{k+1} = F(\boldsymbol{x}_k) + \boldsymbol{w}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{v}$$

where

$$F(\boldsymbol{x}_k) = \boldsymbol{x}_k + \int_{k\Delta t}^{(k+1)\Delta t} \begin{pmatrix} \dot{x} \\ -\ddot{x}_g - c\dot{x} - g(x) \\ \boldsymbol{0} \end{pmatrix}_k dt$$

$\boldsymbol{w}$ is the augmented vector of system noise.

The procedure of the scaled UKF is summarized in **Table 1**.

Table 1 procedure of the scaled UKF

1	**Initialized with** $\hat{\boldsymbol{x}}_0, \hat{\boldsymbol{P}}_0$
2	**For** $k \in \{1, \cdots, N\}$
3	Calculate the sigma points $\boldsymbol{\Psi}_{k-1} = [\hat{\boldsymbol{x}}_{k-1}, \hat{\boldsymbol{x}}_{k-1} + \sqrt{L+\lambda \boldsymbol{P}_{\boldsymbol{x},k-1}}, \hat{\boldsymbol{x}}_{k-1} - \sqrt{L+\lambda \boldsymbol{P}_{\boldsymbol{x},k-1}}]$
4	**The state update process** $\boldsymbol{\Psi}_{\boldsymbol{x},k\mid k-1} = F(\boldsymbol{\Psi}_{\boldsymbol{x},k})$
5	$\hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-} = \sum_{i=0}^{2L} \omega_i^{(m)} \{\boldsymbol{\Psi}_{\boldsymbol{x},k\mid k-1}\}_i$
6	$\boldsymbol{P}_k^{-} = \sum_{i=0}^{2L} \omega_i^{(c)} (\{\boldsymbol{\Psi}_{\boldsymbol{x},k\mid k-1}\}_i - \hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-})(\{\boldsymbol{\Psi}_{\boldsymbol{x},k\mid k-1}\}_i - \hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-})^T$
7	$\boldsymbol{Y}_{k\mid k-1} = \boldsymbol{C}\boldsymbol{\Psi}_{\boldsymbol{x},k\mid k-1}$ $\hat{\boldsymbol{y}}_k = \sum_{i=0}^{2L} \omega_i^{(m)} \{\boldsymbol{Y}_{k\mid k-1}\}_i$
8	**The measurement update process** $\boldsymbol{P}_{\boldsymbol{yy},k} = \sum_{i=0}^{2L} \omega_i^{(c)} (\{\boldsymbol{Y}_{k\mid k-1}\}_i - \hat{\boldsymbol{y}}_k)(\{\boldsymbol{Y}_{k\mid k-1}\}_i - \hat{\boldsymbol{y}}_k)^T$
9	$\boldsymbol{P}_{\boldsymbol{xy},k} = \sum_{i=0}^{2L} \omega_i^{(c)} (\{\boldsymbol{\Psi}_{\boldsymbol{x},k\mid k-1}\}_i - \hat{\boldsymbol{x}}_{\boldsymbol{k}}^{-})(\{\boldsymbol{Y}_{k\mid k-1}\}_i - \hat{\boldsymbol{y}}_k)^T$
10	$\boldsymbol{K}_k = \boldsymbol{P}_{\boldsymbol{xy},k} \boldsymbol{P}_{\boldsymbol{yy},k}^{-1}$
11	$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k^{-} + \boldsymbol{K}_k (\boldsymbol{y}_{\boldsymbol{k}} - \hat{\boldsymbol{y}}_{\boldsymbol{k}})$
12	$\boldsymbol{P}_k = \boldsymbol{P}_k^{-} - \boldsymbol{K}_k \boldsymbol{P}_{yy,k} \boldsymbol{K}_k^T$
13	**end**

2.2 Improved method

In this method, an L_s length section of observation is used as the observation of UKF. It is well known that, unlike linear system, the response of nonlinear system can relate to energy of the input and even response history. This design will help us getting more characteristic of current system to improve the local uniqueness of parameters, at the same time decrease the influence of noise.

This improvement of UKF is similar with an on-line unscented Kalman smoother (UKS)[16]. Kalman Smoother is designed for linear system. It use both history and 'future' observation with positive and negative oriented Kalman process to estimate states of current time point. Regarding nonlinear system, UKS requires other calculation methods to validate backward Kalman process, such as a neural network model.[8]

Different from the UKS, the improved method utilizes the 'future' observations simply as the observation in current estimate. The so called 'current' is not the newest observed time point. The newest observed time point is the last point of the 'future' observations. The current time point is right after the selected section of 'future' observations. The observation estimation in UKF is computed with current first updated states and current estimated parameters. The relationship of these points is illustrated in **Figure 1**.

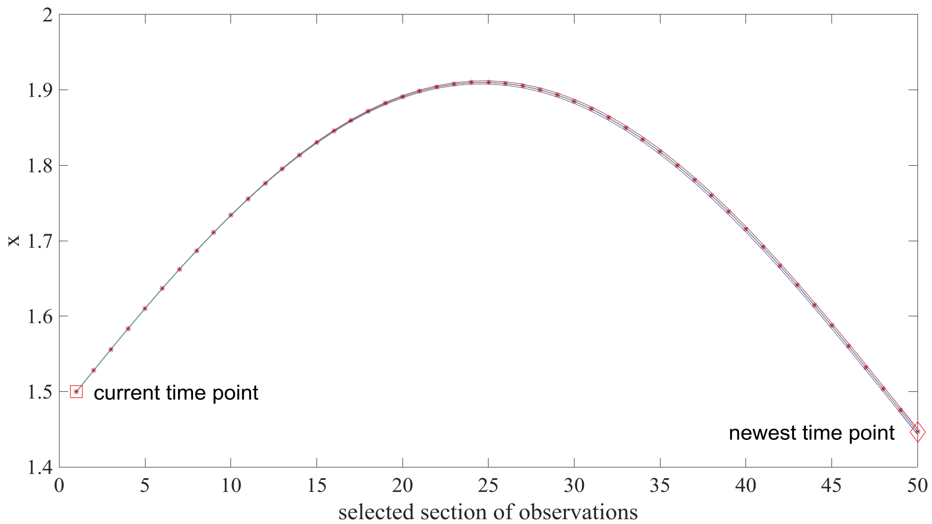

Figure 1 illustration of selected section of observations, current time points marked with□, newest time point is marked with□, several solid lines with different color is the observation estimation generated by sigma points, * is the intact observation calculated by numerical integration

Obviously, the longer the series is, the more stable but the less local the parameter we can get. And if we use all the series points, the computation effort in integration in observation estimate and matrix inverse in P_{xy} calculation will increase explosively with the growth of L_s. To solve this problem, we use $M-1$ random samples of the chosen series together with the point of current time as the observation of UKF. As is proved by the numerical simulation, as long as the identified parameter converges, the random method converges.

The improved method is summarized in **Table 2**

Table 2 the changed procedure in the improved method

Most of the method is the same as table 1, the only change is the 7th step in table 1:

$$\boldsymbol{Y}_{k|k-1} = \boldsymbol{C}\boldsymbol{\Psi}_{\boldsymbol{x},k|k-1}$$

$$\hat{\boldsymbol{y}}_k = \sum_{i=0}^{2L} \omega_i^{(m)} \{\boldsymbol{Y}_{k|k-1}\}_i$$ is replaced with:

Generate M-1 random number $\left[r_1, r_2, r_3, r_4 \cdots\right]$ from (0,1)

$$\boldsymbol{Y}_{k|k-1} = \left[(\boldsymbol{C}\boldsymbol{\Psi}_{\boldsymbol{x},k|k-1})^T, F^{ceil(L_s r_1)}(\boldsymbol{C}\boldsymbol{\Psi}_{\boldsymbol{x},k|k-1})^T, F^{ceil(L_s r_2)}(\boldsymbol{C}\boldsymbol{\Psi}_{\boldsymbol{x},k|k-1})^T, \cdots\right]^T$$

$$\hat{\boldsymbol{y}}_k = \sum_{i=0}^{2L} \omega_i^{(m)} \{\boldsymbol{Y}_{k|k-1}\}_i$$

Where $F^{L_s r_1}(\cdot)$ returns $L_s r_1$ times of $F(\cdot)$ nonlinear transform (integration), $ceil(\cdot)$ returns the larger nearest integer

Why not choose a fix position in the time series?

As the time history of displacement is continuous, observations in fix position of the time series possess the possibility of losing representativeness of the parameters for a while (a certain length of interval), as is shown by **Figure 2**, especially when periodic excitation and evenly distributed fix position is used. During this period, the parameter estimation can possibly lost its way to a wrong position that could never come back in severe nonlinear state space. However, method of random chosen position does not suffer this problem, the position change once a time, which makes a few wrongly estimated step insignificant.

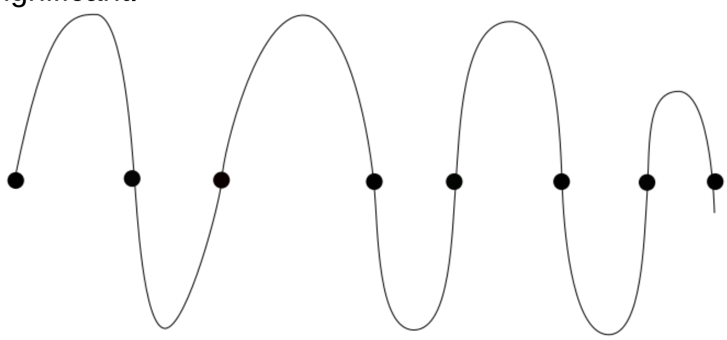

Figure 2 one situation of fix position lose representativeness

The choice of L_s and M

The length L_s should not be too small, a short series may not exhibit enough the characteristics of the system, nor should it be too large, a long series may submerge the local features of the system. A length that can cover a little longer than a quarter of average period of the response is recommended. If the parameters are known to be constant, a larger L_s including few periods is recommended to increase accuracy and robustness of the estimation.

Regarding the choice of M, numerical experiments show that, as small as M=2, which mean a single random point, can offer certain support for parameters to converge to the true value. Users could balance between computation effort and converge rate.

3. Numerical example

In this section, we use a brief Bouc-wen model [17] as the objective of identification. This kind of model was studied in many articles.[13, 18, 19] In these articles, the Bouc-Wen model was often incomplete, that the linear elastic term was often omitted, or the severe nonlinear parameter n was assumed known. The first kind of abbreviation might because that the linear stiffness term and nonlinear stiffness term cannot easily be distinguish by one point observation. The reason of the second kind of abbreviation might be the severe nonlinearity brought by parameter n. Under such severe nonlinear state space, the UKF with one point observation could not easily find the right way to the true parameters. The improved method solved this problem，at the same time, obtained good convergence of either the states and the parameters of the system.

The target nonlinear system was

$$\ddot{x} + c\dot{x} + kx + k_z z = -\ddot{x}_g$$

$$\dot{z} = \rho(1 - \beta sign(\dot{x}) sign(z) | z |^n - (1-\beta) | z |^n)\dot{x}$$

Define:

$$\boldsymbol{x}(1) = x_1 = x;\ \ \boldsymbol{x}(2) = x_2 = \dot{x};\ \ \boldsymbol{x}(3) = x_3 = z;$$

$$\boldsymbol{x}(4) = x_4 = c;\ \ \boldsymbol{x}(5) = x_5 = k;\ \ \boldsymbol{x}(6) = x_6 = k_z;$$

$$\boldsymbol{x}(7) = x_7 = \rho;\ \ \boldsymbol{x}(8) = x_8 = \beta; \boldsymbol{x}(9) = x_9 = n;$$

The state space representation of the dynamic system is

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\ddot{x}_g - x_4 x_2 - x_5 x_1 + x_6 x_3$$

$$\dot{x}_3 = x_7(1 - x_8 sign(x_2) sign(x_3) | x_3 |^{x_9} - (1 - x_8) | x_3 |^{x_9}) x_2$$

$$\dot{x}_4 = 0$$

$$\dot{x}_5 = 0$$

$$\dot{x}_6 = 0$$

$$\dot{x}_7 = 0$$

$$\dot{x}_8 = 0$$

$$\dot{x}_9 = 0$$

The excitation was a composition of 4 sinusoidal with frequency 0.4 Hz、1Hz、1.5 Hz、1.8 Hz and amplitude 3、5、3、3 respectively. The excitation was designed to fully stimulate the characteristic of this hysteretic system. The sample rate was 100Hz. The integration method was the explicit RK4. The true value of the parameters were $c = 0.6,\ k = 40,\ k_z = 20,\ \rho = 3,\ \beta = 0.6,\ n = 2$. The initial value of the improved UKF were arbitrarily chose, which was set as $c=0.2,\ k=30,\ k_z=20,\ \rho=5,\ \beta=0.3,\ n=3$.

The identified parameters by intact signal (with no noise contaminated) are shown in **Figure 3**, the estimated states are illustrated in **Figure 4**. L_s and M were set as 50 and 8 respectively. From **Figure 4~Figure 10** contaminated signal with signal to noise ratio (SNR) from 50Db to 35Db were used. L_s and M were set as100 and 16 respectively.

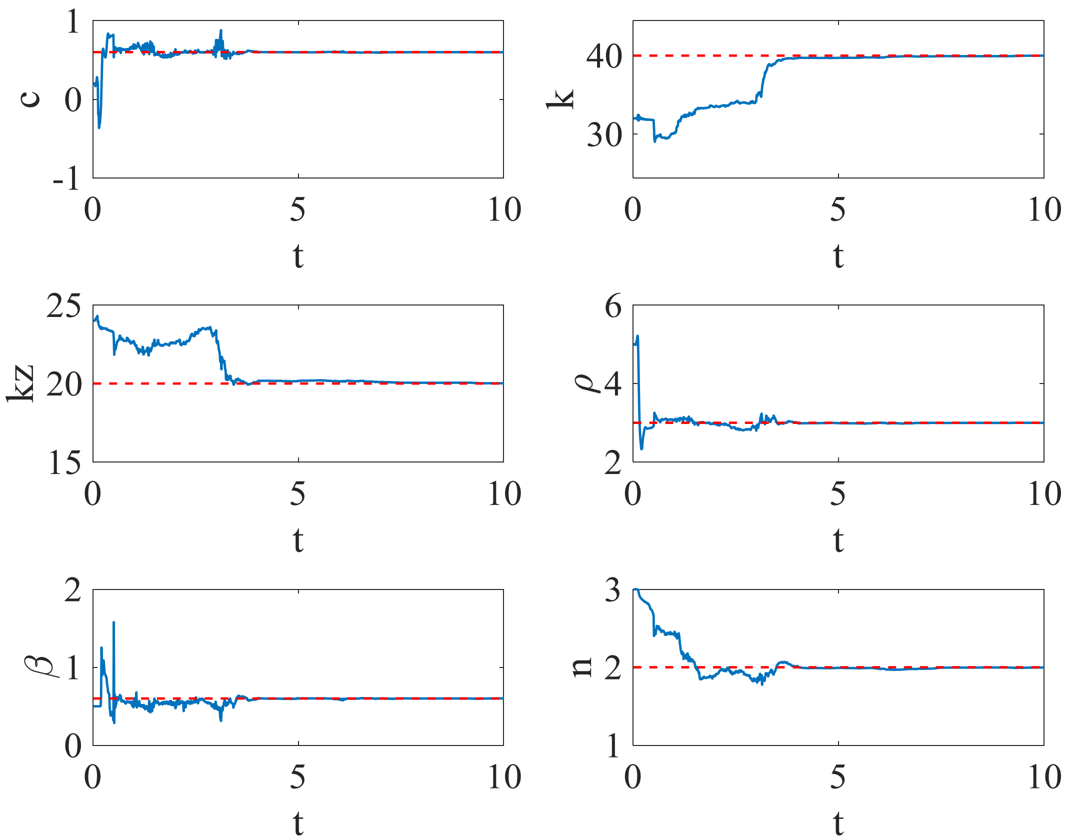

Figure 3 identification results of parameter using intact signal (the right dashed line stands for the true value of parameter, the blue solid line stands for the identified value, the same line style is used in Fig. 4,6,8)

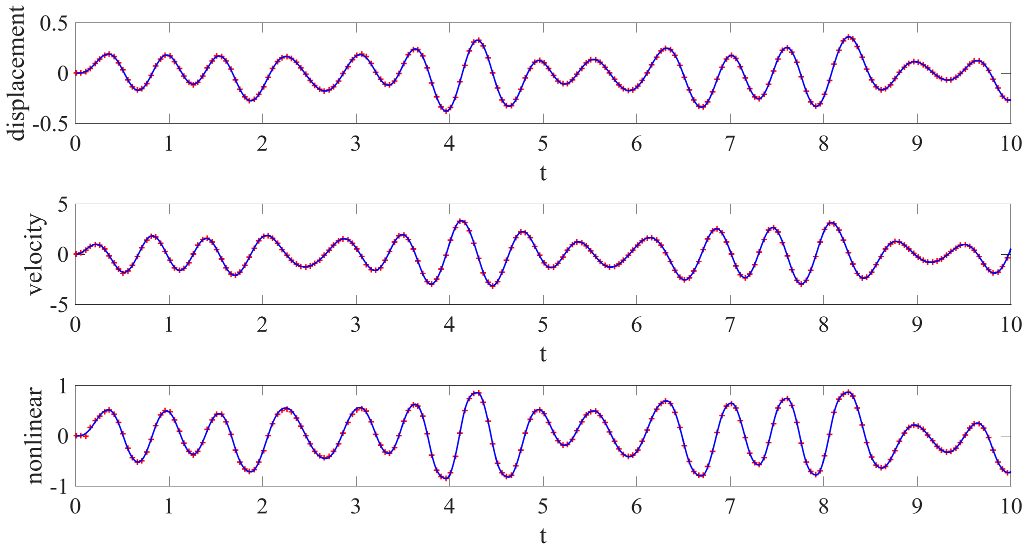

Figure 4 identification results of states using intact signal(the right '+' mark stands for the identified states, the blue solid line stands for the true value, the same mark is used in Fig. 5,7,9)

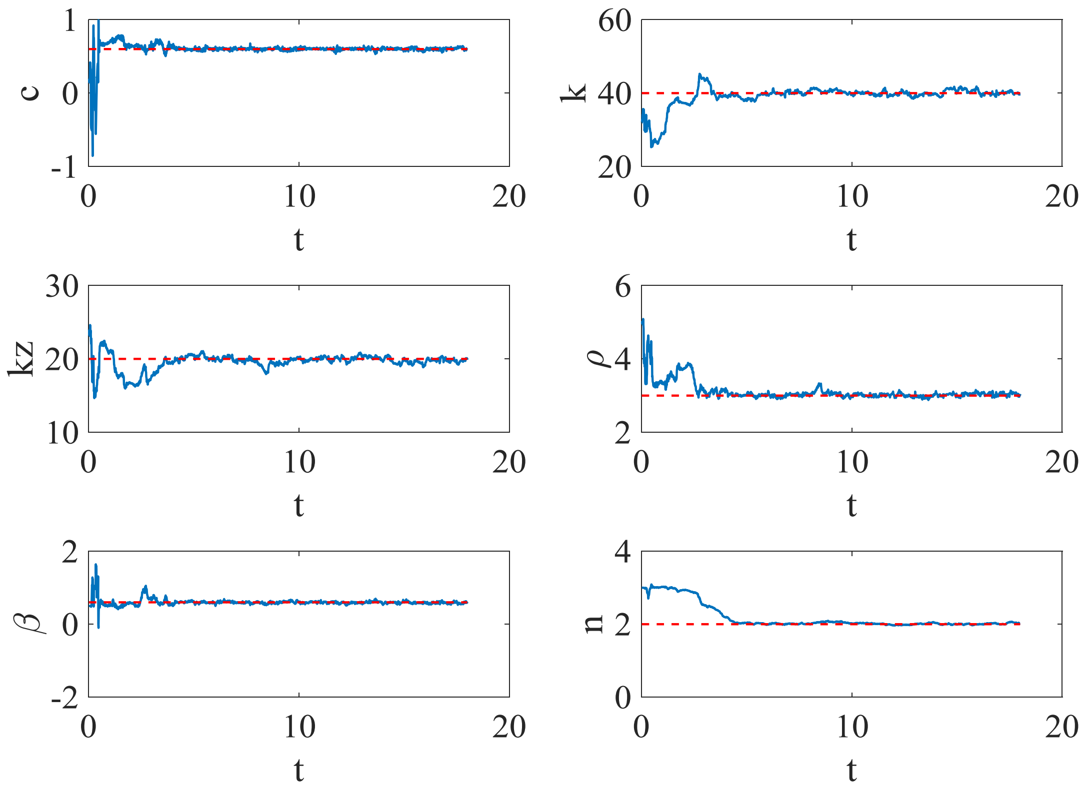

Figure 5 identification results of parameter using SNR 50db signal

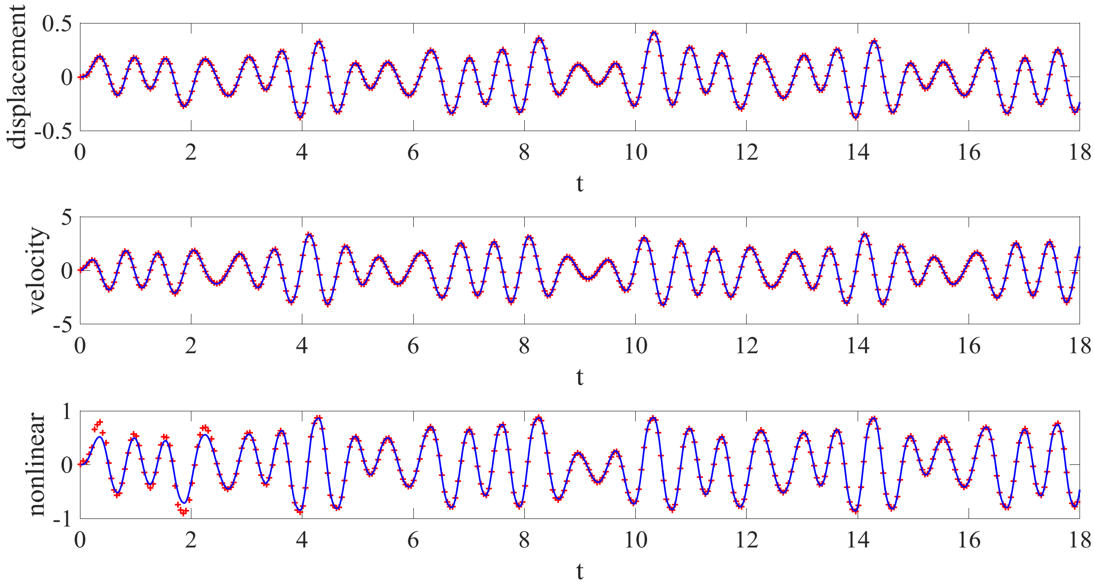

Figure 6 identification results of states using SNR 50db signal

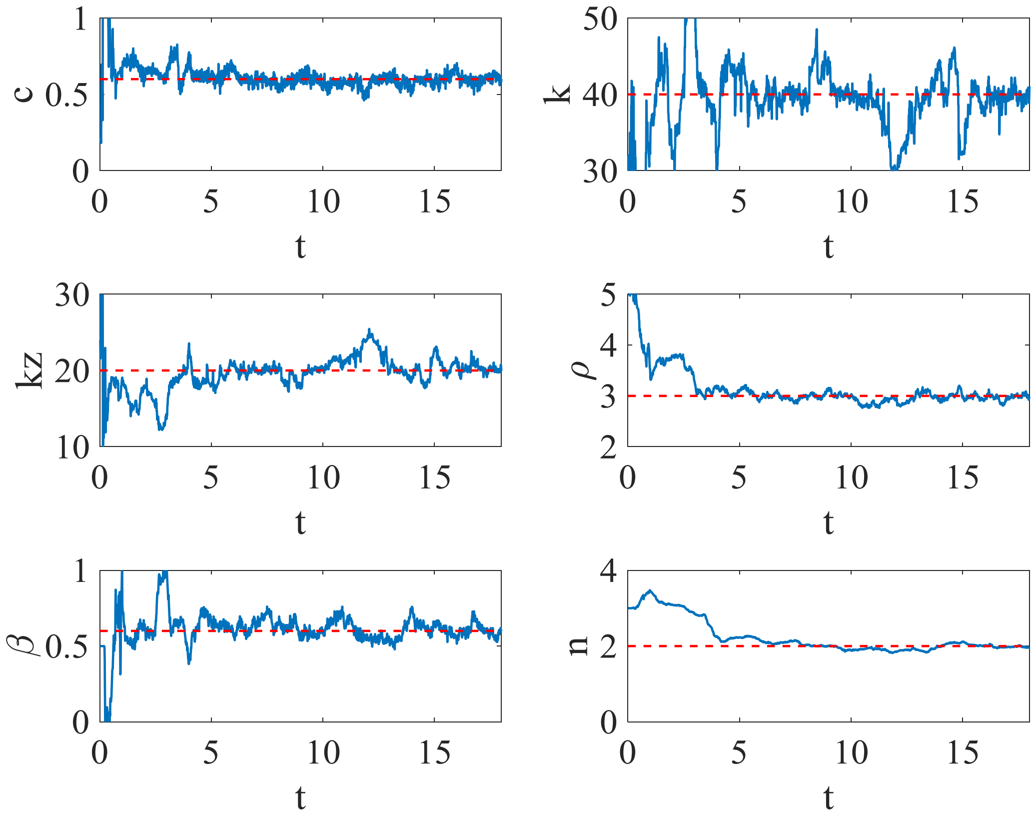

Figure 7 identification results of parameter using SNR 40db signal

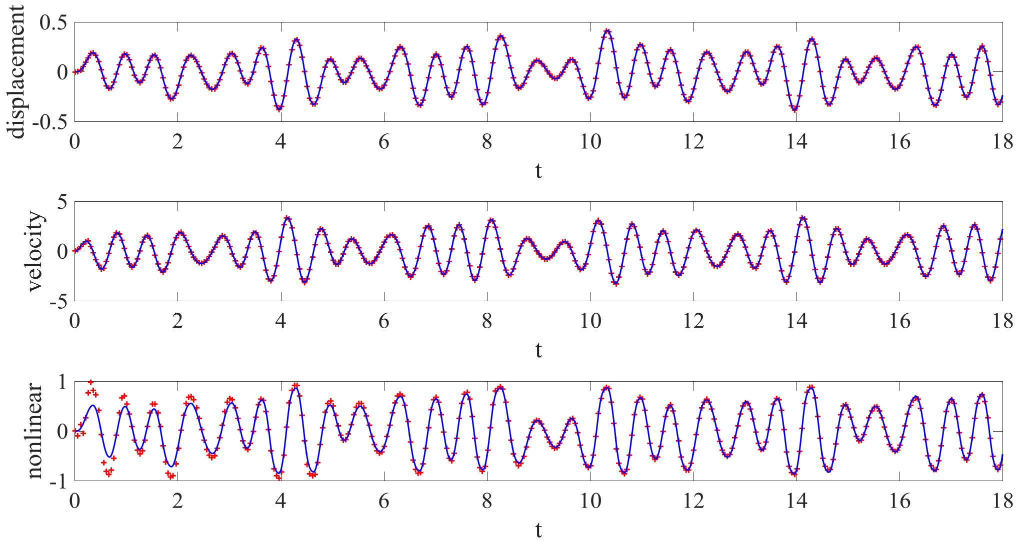

Figure 8 identification results of states using SNR 40db signal

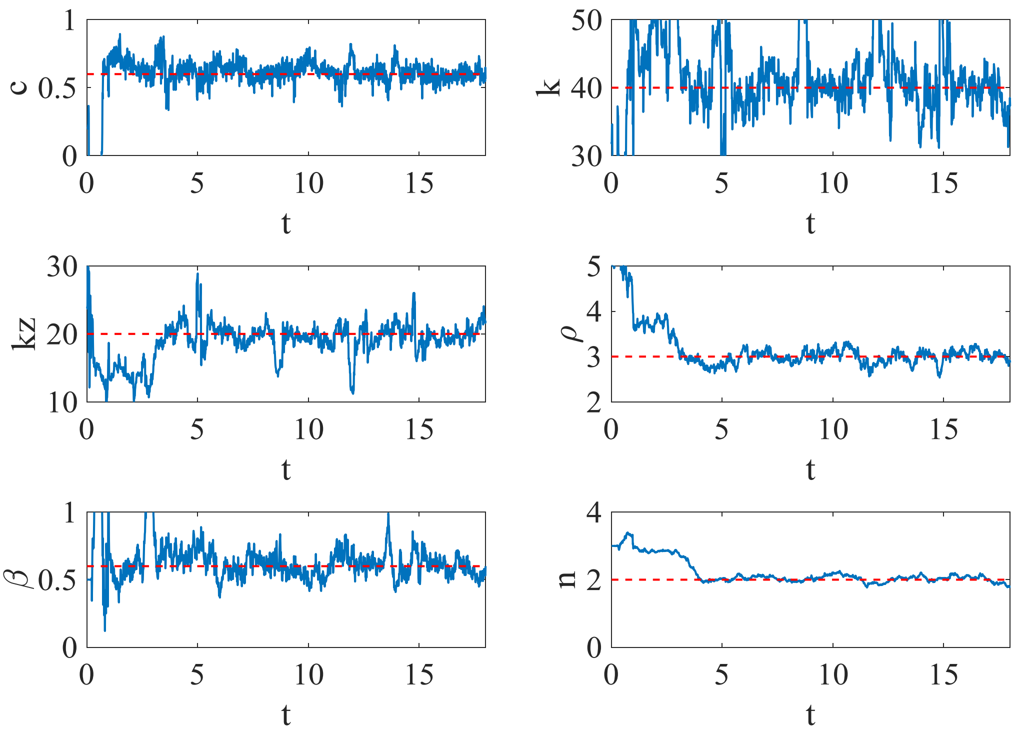

Figure 9 identification results of parameter using SNR 35db signal

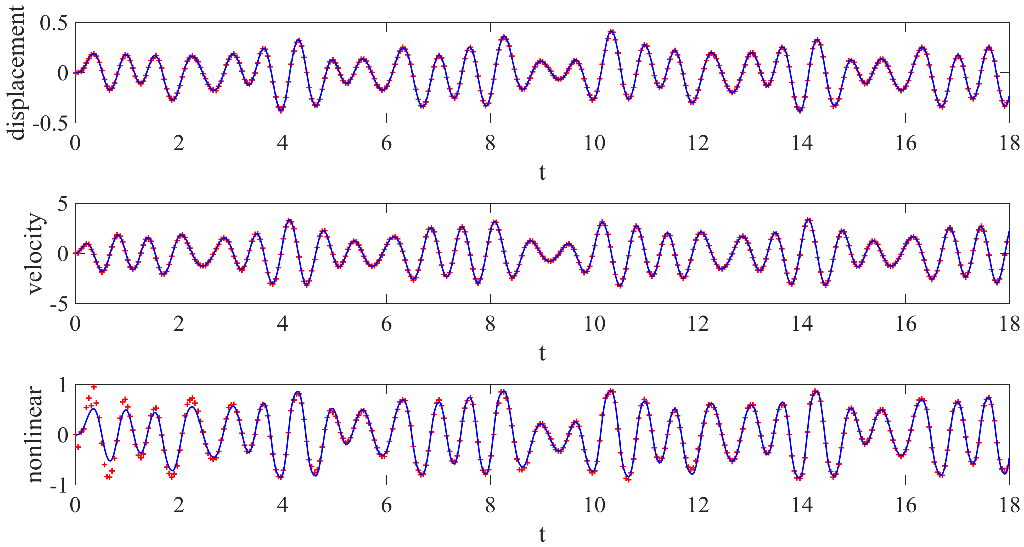

Figure 10 identification results of states using SNR 35db signal

As it is shown in **Figure 3**, parameters converge quickly within 5 seconds. When signal is little contaminated, parameters is well identified in **Figure 5**. As the noise level increase, the estimation of parameters become worse. However the parameters obtained by this method can still oscillate tightly around the true value.

4. Conclusion

The improved UKF method successfully identified the parameters and states form the contaminated observations, which shows the effectiveness of this method in system identification against complicated model and noise.

However, this method is not a real on-line identification method, the time delay depends on the chosen length of the observation. This problem will be studied in subsequent works. The application in real environment is another problem to solve.

Reference

[1] M. S G, A. P A. Applications of Kalman Filtering in Aerospace 1960 to the Present [Historical Perspectives][J]. IEEE Control Systems, 2010,30(3):69-78.

[2] SGMcDaniel. Global positioning systems, inertial navigation, and integration[M]. Wiley, 2007.

[3] Costa P J. Adaptive model architecture and extended Kalman-Bucy filters[J]. Aerospace & Electronic Systems IEEE Transactions on, 1994,30(2):525-533.

[4] Prasad G, Irwin G W, Swidenbank E, et al. Plant-wide predictive control for a thermal power plant based on a physical plant model[J]. Control Theory and Applications, IEE Proceedings -, 2000,147(5):523-537.

[5] Arulampalam M S, Maskell S, Gordon N, et al. A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking[J]. IEEE Trans Signal Processing,

2002,50(174):174-188.
[6] Julier S J, Uhlmann J K. Unscented filtering and nonlinear estimation[J]. Proceedings of the IEEE, 2004,92(3):401-422.
[7] Arasaratnam I, Haykin S. Cubature Kalman Filters[J]. IEEE Transactions on Automatic Control, 2009,54(6):1254-1269.
[8] Haykin S. Kalman Filtering and Neural Networks[J]. Adaptive & Learning Systems for Signal Processing Communications & Control, 2001:170-174.
[9] Wan E A, Nelson A T. Nonlinear estimation and modeling of noisy time-series by dual Kalman filtering methods[J]. Thesis Ogi Sch.of Sci.and, 2000.
[10] Gove J H, Hollinger D Y. Application of a dual unscented Kalman filter for simultaneous state and parameter estimation in problems of surface-atmosphere exchange[J]. Journal of Geophysical Research Atmospheres, 2006,111(D8):21.
[11] Sitz A, Schwarz U, Kurths J, et al. Estimation of parameters and unobserved components for nonlinear systems from noisy time series.[J]. Physical Review E Statistical Nonlinear & Soft Matter Physics, 2002,66(1 Pt 2):-.
[12] Wu M, Smyth A W. Application of the unscented Kalman filter for real-time nonlinear structural system identification[J]. Structural Control & Health Monitoring, 2007,14(7):971-990.
[13] Xie Z, Feng J. Real-time nonlinear structural system identification via iterated unscented Kalman filter[J]. Mechanical Systems & Signal Processing, 2012,28(2):309-322.
[14] Astroza R, Ebrahimian H, Conte J P. Material Parameter Identification in Distributed Plasticity FE Models of Frame-Type Structures Using Nonlinear Stochastic Filtering[J]. Journal of Engineering Mechanics, 2014,141(5).
[15] Simon D. Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches[M]. Wiley-Interscience, 2006.
[16] Ikhouane F, Rodellar J. Systems with Hysteresis: Analysis, Identification and Control using the Bouc-Wen Model[M]. 2007.
[17] Wu M, Smyth A W. Application of the unscented Kalman filter for real-time nonlinear structural system identification[J]. Structural Control and Health Monitoring, 2007,14(7):971-990.
[18] Yuen K V, Kuok S C. Online updating and uncertainty quantification using nonstationary output-only measurement[J]. Mechanical Systems & Signal Processing, 2015,66.