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Damage-based Strength Reduction Factor for Sequence-type Ground Motions

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ABSTRACT

This paper investigates the strength reduction factor of single-degree-of-freedom system subjected to the mainshock-aftershock sequence-type ground motions. Records of mainshock-aftershock earthquakes were collected and classified into four groups according to the criteria specified in the Code for Seismic Design of Buildings in China. Based on the nonlinear time-history analysis of single-degree-of-freedom inelastic systems, the effects of period, ductility factor, damage index and aftershock have been studied statistically. The results indicate that the aftershock ground motion has a significant influence on strength reduction factors, and the damage-based strength reduction factor is about 0.6-0.9 times that of the ductility-based strength reduction factor. Finally, the empirical expression of strength reduction factors was established by statistical mean method and regression analysis.

1. INTRODUCTION

According to statistics, about 88% of strong earthquakes are accompanied by aftershocks (Wu 1987). An aftershock is defined as a smaller earthquake following the mainshock, which is the largest earthquake in the sequence. Structural damage caused by the mainshock is further aggravated under aftershocks and can lead to structural collapse. The 2010 New Zealand and the 2015 Nepal earthquakes (Qu 2015) experienced both mainshock and aftershock ground motions, and are good examples of why sequence-type ground motions are important factors in the structural design stage. In recent years, researchers have explored these factors. Some researchers focus on the impact of sequence-type ground motions on inelastic spectra (Amadio 2003, Wen 2011, Hatzigeorgiou 2009 2010). These studies also consider strength reduction factor spectra, damage spectra, ductility factor spectra, etc. Other researchers focus on the changes in structural response under sequence-type ground motions(Ou 1994, Li 2007,

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Zhu 2012), which includes peak displacement, damage level, etc. All results show increased structural damage after sequence-type ground motions.

Current seismic design principles include analysis of a structure's elastic-plastic behavior under rare earthquakes. The design strength of most structures is generally much lower than the minimum strength required to maintain the elastic stage under strong earthquakes. The reduction factor is often used to reduce the elastic strength demand and thereby obtain the elastic-plastic strength demand of a structure. Theoretical analysis and experimental studies of strength reduction factors have found that the ductility factor has a significant effect on the strength reduction factor. The displacement ductility factor helps to assess the extent of structural damage. To obtain the ductility-based strength reduction factor R_μ, begin with Eq. (1):

$$R_\mu = \frac{F_e}{F_{y,\mu}} = \frac{F_y(\mu = 1)}{F_y\left(\mu = \mu_i\right)} \tag{1}$$

where $F_y(\mu = 1)$ is the minimum required strength of the structure to maintain the elastic stage under earthquake ground motions, which is the maximum load F_e that elastic structures can withstand. $F_y(\mu = \mu_i)$ is the yield strength demand F_y of the structure when the ductility factor is μ_i.

In addition to displacement ductility, cumulative damage of structures resulting from the reciprocating load also plays an important role in structural damage. To understand the effect of the reciprocating load, modify the ductility factor to indirectly consider the influence of cumulative damage (Fajfar 1992, Cosenza 2009), introducing an equivalent ductility factor, and a weighted ductility factor. There are also studies that directly consider the impact of cumulative damage on the strength reduction factor (Hou 2013). Strength demand can be determined at a particular level of performance through the introduction of a reasonable structure damage model, thereby obtaining the direct damage-based strength reduction factor R_D, which can be written as Eq. (2):

$$R_D = \frac{F_e}{F_{y,D}} = \frac{F_y(\mu = 1,\ D = 0)}{F_y\left(\mu = \mu_i, D = D_j\right)} \tag{2}$$

where $F_y(\mu = 1, D = 0)$ is the minimum required strength of the structure used to maintain elastic stage under earthquake ground motions, i.e. the maximum load F_e that the elastic structures can withstand. $F_y(\mu = \mu_i, D = D_j)$ is the yield strength demand F_y of the structure when the ductility factor is μ_i and the structural damage state is D_j.

Existing damage-based strength reduction factors do not reflect the requirements of Chinese design response spectra, according to site classification standards. Therefore, applying the above strength reduction factor directly raises questions concerning inelastic spectra. This research is based on as-recorded ground motions, wherein sites were divided into different categories according to Chinese seismic design code. The elastic-plastic time history analysis for the single-degree-of-freedom system came from data based on different periods. Hence, comparisons were made of damage-based strength reduction factors under both single and sequence-type earthquake ground motions. Impact of ductility factor, damage index and other factors pertaining to strength reduction factors were included in the final analysis. All analyses led to the development of a new empirical formula which helps to measure and define the damage-based strength reduction factor.

2. SEQUENCE-TYPE EARTHQUAKE GROUND MOTIONS AND SITE CATEGORIES CLASSIFICATION

Seismic input is a prerequisite to assessing the seismic response of a structure. Due to the inherent stochastic characteristics of ground motions, large quantities of natural earthquake responses are essential. Records of two sequential earthquake ground motions were used based on Eq. (1) successive ground motion records from the same station and Eq. (2) recording mainshocks with peak ground accelerations (PGAs) of 0.1g or more.

Because of the small number of strong motion monitoring stations in China, data observations are limited. Currently, researchers must also include ground motion data from the United States, Japan and other countries. However, the principles of site classification in the seismic codes of China and the United States are very different. To make better use of these differences, (Guo 2011) et al. compared and analyzed site classification indicators in the seismic design codes of China and the United States. Each site is divided according to the equivalent shear wave velocity V_{30} and the Chinese seismic design code, whereupon the regression formula is given as Eq. (3):

$$\ln(V_{20}) = 0.4109 + 0.908\ln(V_{30}) \tag{3}$$

Site categories are obtained by site conversion in China, as shown in Table 1. Using the 256 ground motions from the Pacific Earthquake Engineering Research Center (PEER), in accordance with the above principles, sequence-type ground motion were built and Chinese site categories were divided according to conversion relations, as shown in Table 2. Due to the small number of actual mainshock–aftershock sequence-type ground motions for Classes I and IV sites, this research included ground motion records for Class II and Class III sites. Meanwhile, the amplitude of the aftershock was modulated so that the aftershock peak acceleration values were the same as the mainshock peak acceleration values, that is, PGAas / PGAms = 1.

Table 1 Site Conversions

V_{30}	Site category	V_{20}
V_{30}>500m/s	Class I	V_{20}>596m/s
500m/s > V_{30}>250 m/s	Class II	596m/s > V_{20}>278 m/s
250m/s > V_{30}>150 m/s	Class III	278m/s > V_{20}>158 m/s
150m/s > V_{30}	Class IV	158m/s > V_{20}

Table 2 Number of recorded sequence-type ground motions used in this research

Name of the earthquake	Time	Magnitude of mainshock	Site category	
			Class II	Class III
Hollister	1961/04/09 07:23	5.6		1
Managua, Nicaragua	1972/12/23 06:29	6.2	2	
Imperial Valley	1979/10/15 23:16	6.5		26
Livermore	1980/01/24 19:00	5.8		1
Mammoth Lakes	1980/05/25 16:34	6.1	6	
Mammoth Lakes(1)	1983/01/07 01:38	5.3	2	
Coalinga	1983/07/22 02:39	6.4		2
Chalfant Valley	1986/07/20 14:29	5.8		3
Whittier Narrows	1987/10/01 14:42	6.0	16	4
Superstition Hills	1987/11/24 05:14	6.22		2
Northridge	1994/01/17 12:31	6.7	22	5
Chichi	1999/9/20	7.6	93	69
		Total	143	113

3. PERFORMANCE LEVEL DEFINITIONS AND LIMITS

Structure damage takes on various forms under an earthquake, and the extent of the structural damage may not be fully reflected only by maximum deformation. This is why reasonable indicators must be used to assess the extent of structural damage. At present, the international research community agrees that the maximum deformation of a structure and its hysteretic energy are the main factors of structural damage. This agreement, however, has been presented in a variety of two-parameter damage models. This paper uses the Park-Ang (Park 1985) model to assess the damage index, which can be written as Eq. (4):

$$D=\frac{\mu_m}{\mu_u}+\beta\frac{E_h}{F_y\mu_u x_y} \tag{4}$$

where D is the damage index; μ_m is the ductility factor when the structure reaches the maximum elastic-plastic deformation under ground motions; μ_μ is the ductility factor when the structure fails under monotonic loading; F_y is the yield strength; E_h is the cumulative hysteretic energy dissipation under ground motions; β is the energy factor, and it is 0.15 for frame structures.

Based on Park-Ang damage model, through the damage investigation of the actual structure, the structural damage under earthquake ground motion is divided into five performance levels as defined by (Ou 1999) et al. Each performance level corresponds to a damage index range as shown in Table 3.

4. ANALYSIS METHOD AND STRUCTURAL PARAMETERS

The equation of motion of a nonlinear single-degree-of-freedom system under earthquake ground motion is:

$$m\ddot{x} + c\dot{x} + f_s = -m\ddot{x}_g \tag{5}$$

where c is the damping coefficient; f_s is the restoring force of the structure; x is the relative displacement, and x_g is the ground displacement.

According to the definition of the strength reduction factor in Eq. (2) and Eq. (5), when the elastic vibration cycle, damping ratio and restoring force model of the single-degree-of-freedom system are known, it is able to iterate by numerical analysis the yield strength $F_y(\mu=\mu_i, D=D_j)$ for each input ground motion, and the specific period and target displacement, until the calculated displacement ductility factor μ_i and damage index D_j are within the allowable accuracy range, thus obtaining the yield strength $F_y(\mu=\mu_i, D=D_j)$ to calculate the strength reduction factor R_D. A series of strength reduction factors R_D within single-degree-of-freedom system in different ductility factors μ_i and damage indexes D_j can be obtained by calculating different periods and ground motions, which constitute the strength reduction factor spectra. The calculation steps are shown in Fig. 1, where a specific numerical analysis uses the Newmark-β method, and the relative error is controlled under 1%.

In this paper, a single-degree-of-freedom system is the research object, and the hysteretic model is the ideal elastic-plastic model, because of its simple constitutive relationship. Meanwhile, some characteristics of the structural system can be reflected by the structural response under earthquake ground motions. The vibration period of SDOF systems is from 0.1s to 6s with an interval of 0.1s, thus, a total of 60 different periodic points are calculated, with damping ratio of 5%, considering ductility factors of 2, 3, 4, 5, 6 and damage indexes as 0.2, 0.4, 0.6, 0.9, and 1.0, respectively.

Table 3 Damage index ranges for different performance levels

Performance level	Mainly intact	Slightly damaged	Moderate damaged	Serious damaged	Collapsed
Damage index	0<*D*<0.2	0.2<*D*<0.4	0.4<*D*<0.6	0.6<*D*<0.9	0.9<*D*

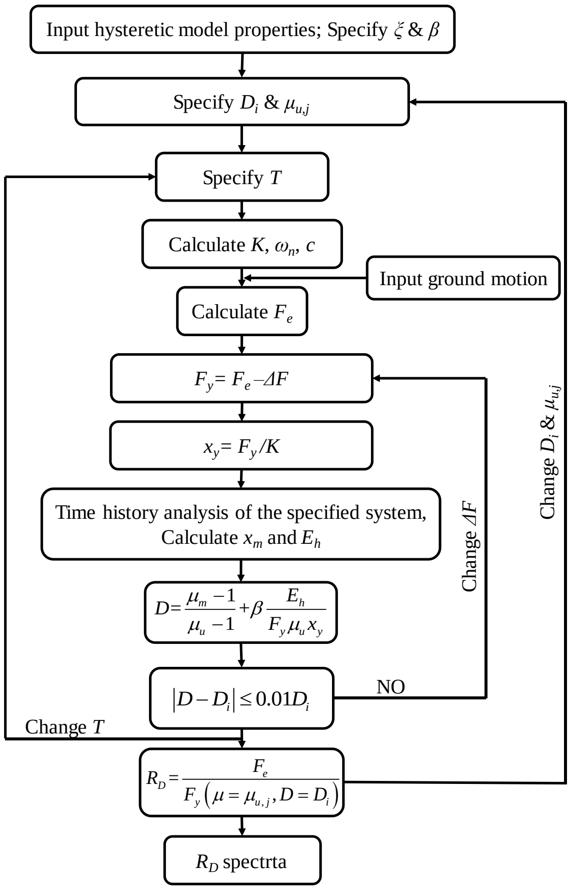
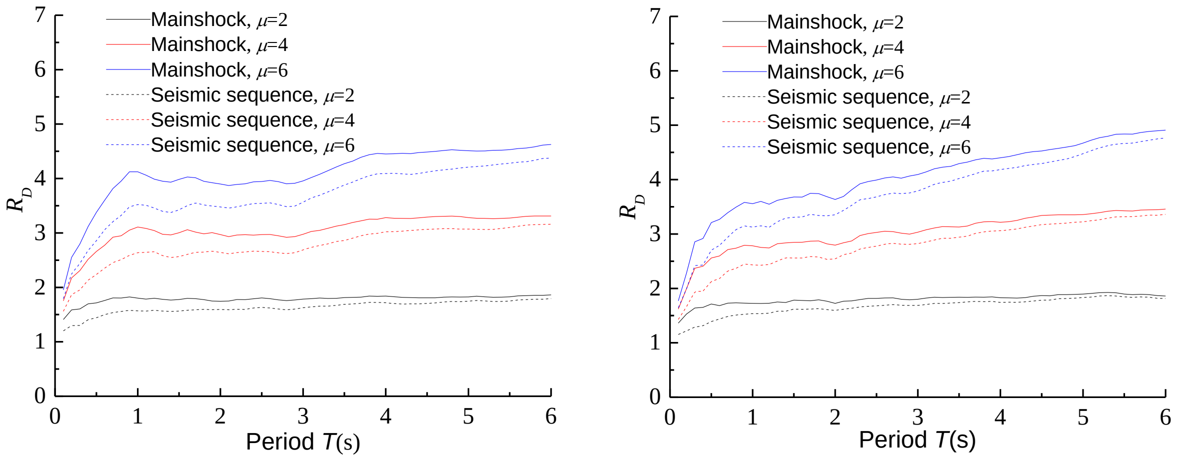


Fig. 1 Flowchart for computation of the strength reduction factor

5. THE AVERAGE STRENGTH REDUCTION FACTOR SPECTRA

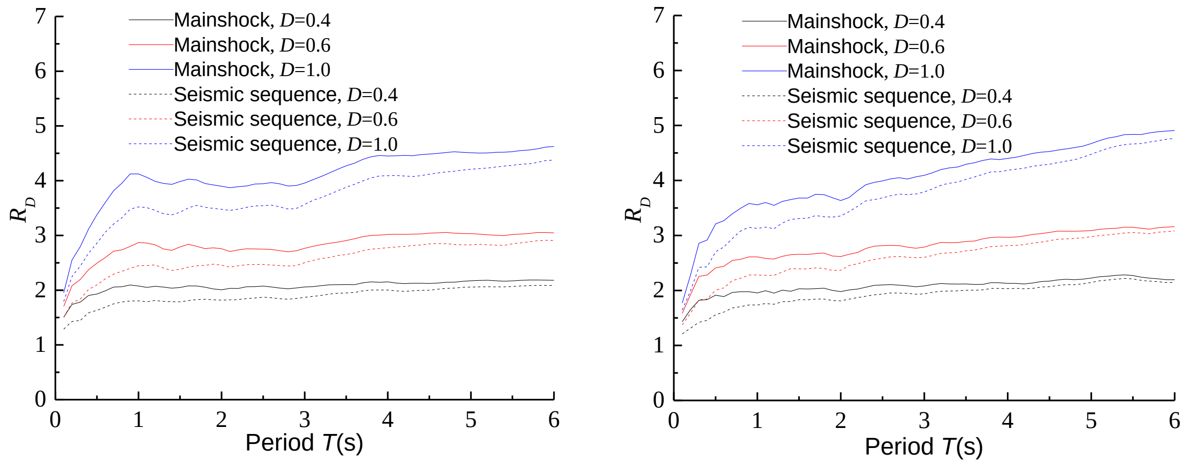
Based on 512 selected ground motions (from single earthquakes and sequence-type earthquakes), a total of 768,000 working conditions with 60 vibration periods, five ductility factors and five damage indexes are calculated, to obtain strength reduction factors. The statistical analyses of those factors were used to obtain the corresponding average R_D spectra and the coefficient of variation. Due to space limitations, only part of the results can be presented in this paper. The average strength reduction factor is shown in Fig. 2 and Fig. 3. Analysis of the basic characteristics of strength reduction factors can be summarized as:

(1) For different ductility factors and damage indexes, strength reduction factor spectrum is almost the same. That is, in the short period stage (0-1s), with the increasing period, the strength reduction factor dramatically increases; in the long period (1-6s), the strength reduction factor changes slightly. Because the structural rigidity changes dramatically in the short period, especially when the period approaches zero, the structural rigidity tends toward infinity. A small change of strength reduction factor may cause a large change of strength, so in the short period, the strength reduction factor decreases sharply with the decrease of the time period.



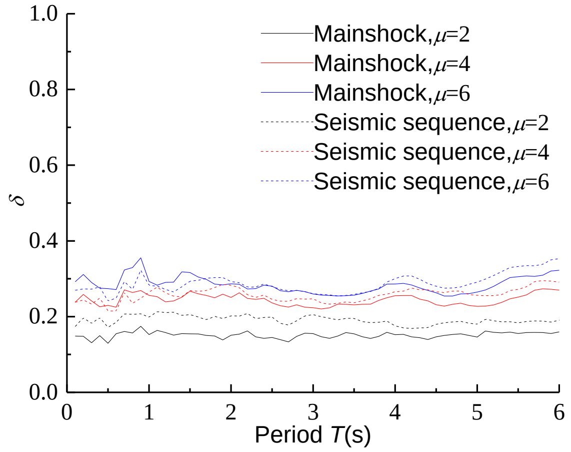
(a) Site class II (*D*=1.0)   (b) Site class III (*D*=1.0)

Fig. 2 Influence of ductility on R_D factor

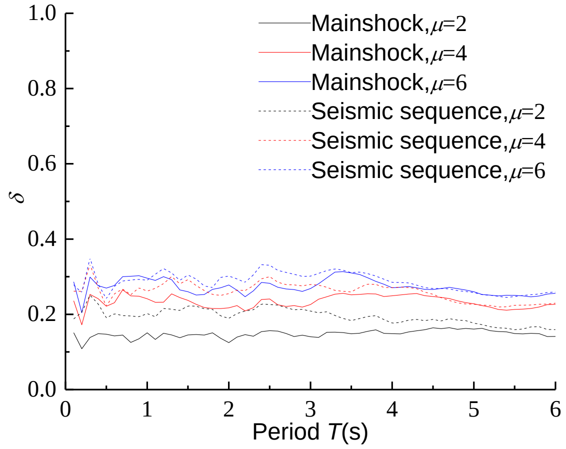


(a) Site class II (*μ*=6)   (b) Site class III (*μ*=6)

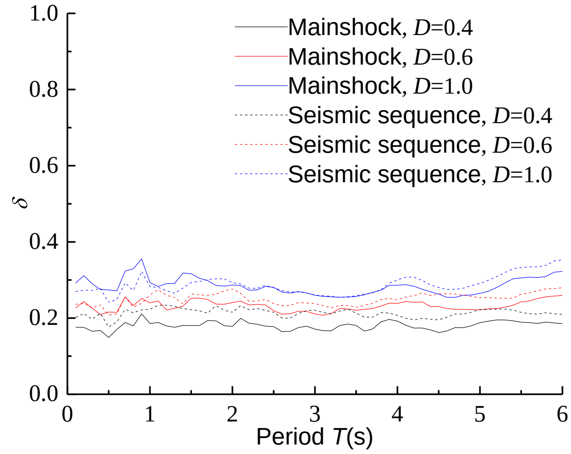
Fig. 3 Influence of damage index on R_D factor



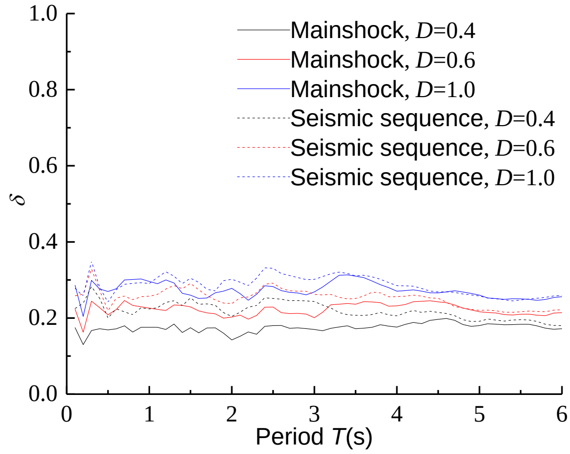
(a) Site class II (D=1.0)



(b) Site class III (D=1.0)



(c) Site class II (μ=6)



(d) Site class III (μ=6)

Fig. 4 The COVs of mean R_D factor

(2) In the same period, under certain damage indexes, the strength reduction factor increases with the increase of the ductility factor, that is, the structural strength demand decreases with the increase of the ductility factor. When the structure has sufficient ductility, it can withstand a certain degree of damage caused by the earthquake.

(3) In the same period, under a certain ductility factor, the strength reduction factor increases with the increase of the target damage index, that is, under the same external load, the greater the structural target damage is, the smaller the strength demand is.

(4) The strength reduction factor calculated under the sequence-type earthquake is less than that of a single earthquake, especially in the short period stage, with a maximum difference of 22%. The structural design strength demand under the sequence-type earthquake is greater than the strength demand under a single earthquake. Aftershocks significantly affect the structural strength demand.

To reflect the degree of dispersion in the strength reduction factor spectra, calculate the coefficients of variation of the corresponding mean strength reduction factor spectrum, as shown in Fig. 4. As can be seen by comparing, for a particular period, the coefficient of variation increases with the increase of damage index and ductility factor. The maximum coefficient of variation of the mean strength reduction factor calculated under each group site condition does not exceed 40%, which reflects the randomness and discreteness of the ground motion to a certain degree.

6. REGRESSION ANALYSIS

6.1.The establishment of the R_D spectrum

In order to facilitate structural seismic design applications, based on the above analysis results, the empirical formula is proposed to estimate the strength reduction factor. Compared with the expression of the traditional ductility strength reduction factor R_μ, the strength reduction factor R_D considers damage by adding the damage index, thus, the R_D empirical formula is:

$$R_D = R_D(T, D, \mu_u) \tag{6}$$

where *T, D,* μ_μ are natural vibration period, damage index, and displacement ductility factor, respectively. According to the actual stress state of the structure, the strength reduction factor expression must meet the following four conditions: $R_D(T \to 0, D, \mu_u) = 1$ (1), $R_D(T, D = 0, \mu_u) = 1$ (2), $R_D(T, D, \mu_u = 1) = 1$ (3), and $R_D(T \to \infty, D, \mu_u) \to R_D^c$ (4)

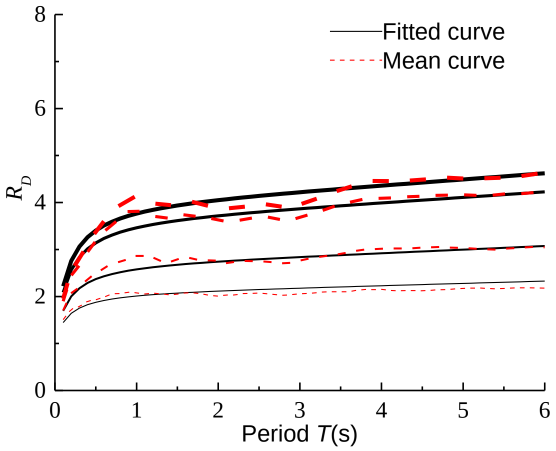
According to the above conditions and the influence of the structure period, it is the damage index, and the displacement ductility factor on the strength reduction factor that are actually calculated. The simplified model of the strength reduction factor, obtained by fitting, is:

$$R = 1 + D^{1.1} \cdot [0.2(\mu - 1)]^{0.85} \frac{aT + bT^2}{1 + cT + dT^2} \tag{7}$$

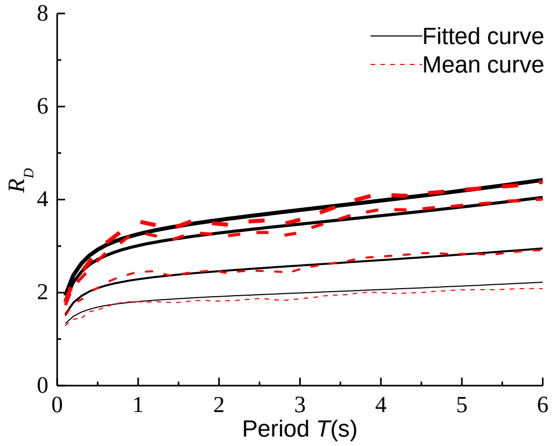
where a, b, c, d are fitting parameters, relative to site classification and earthquake categories, as shown in Table 4. Comparisons between the fitting curve obtained by the fitting formula and the average strength reduction factor curve actually calculated are shown in Fig. 5, and Fig. 6.

Table 4 The value of *a~d*

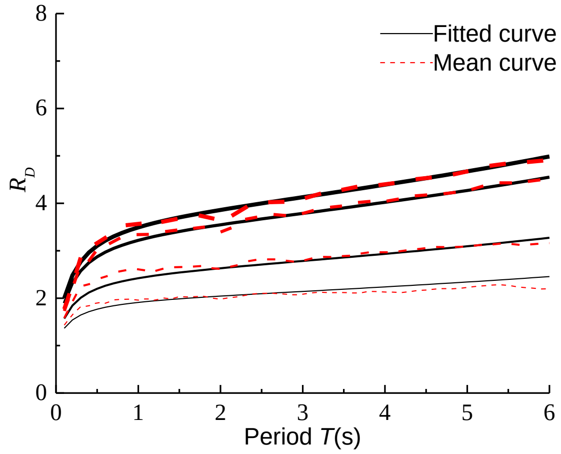
	Parameter	*a*	*b*	*c*	*d*
Class II	Single earthquake	19.25	-0.54	5.97	-0.56
	Sequence-type earthquake	13.66	-0.71	5.14	-0.43
Class III	Single earthquake	15.32	-0.12	5.41	-0.32
	Sequence-type earthquake	10.41	0.67	4.48	-0.15



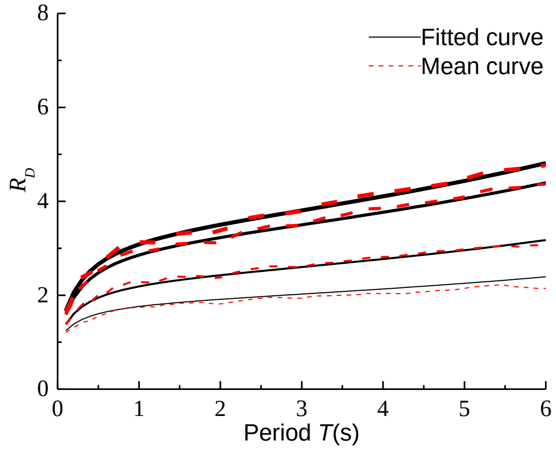
(a) Site class II (mainshock)



(b) Site class II (seismic sequence)

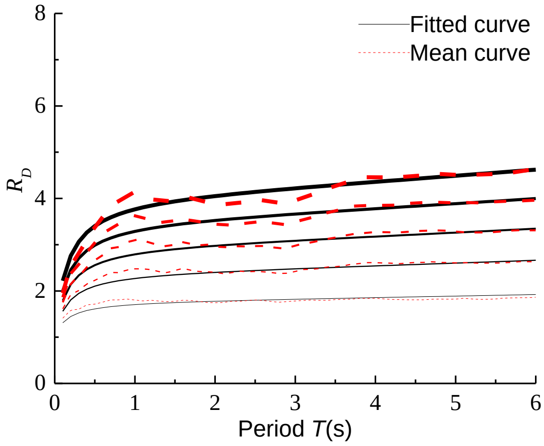


(c) Site class III (mainshock)

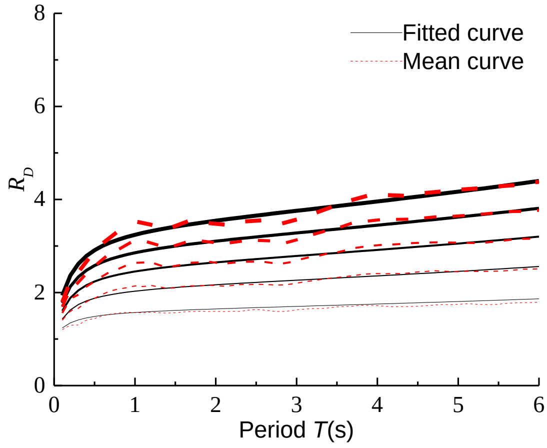


(d) Site class III (seismic sequence)

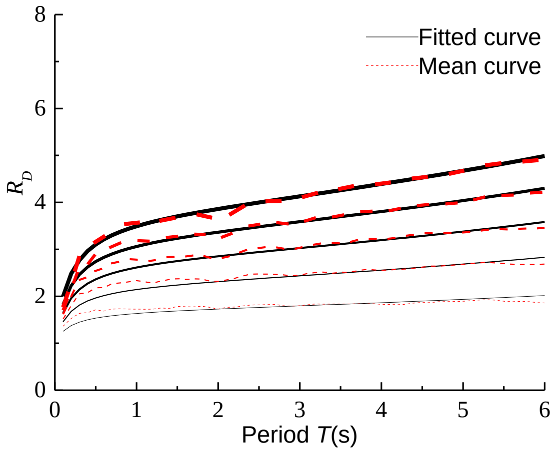
Fig. 5 Comparison of the computed R_D spectra with the original spectra (μ=6)



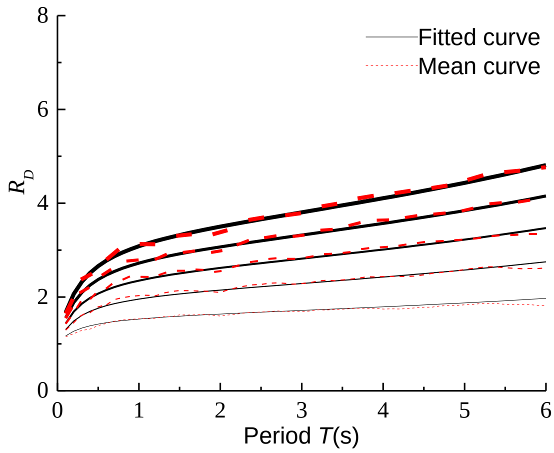
(a) Site class II (mainshock)



(b) Site class II (seismic sequence)



(c) Site class III (mainshock)



(d) Site class III (seismic sequence)

Fig. 6 Comparison of the computed R_D spectra with the original spectra (D=1.0)

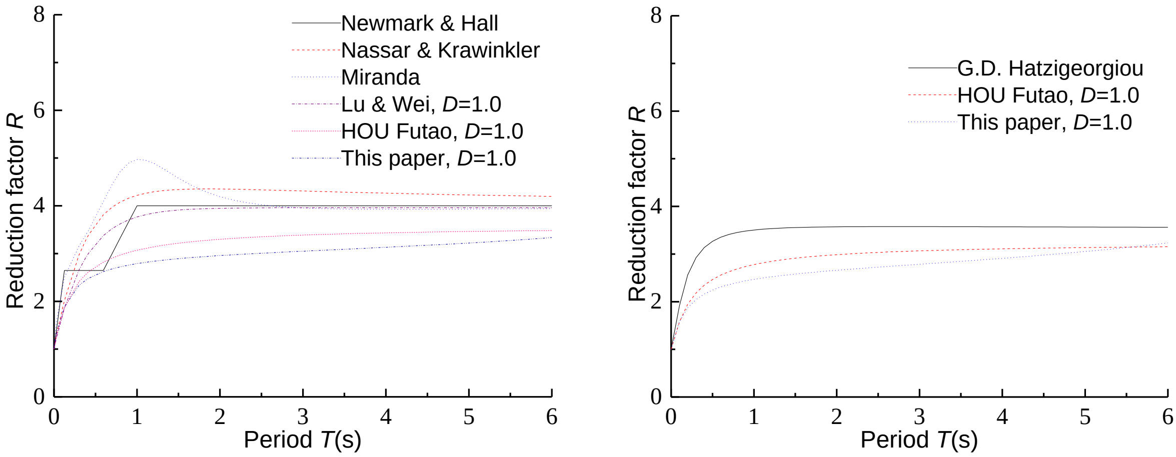


Fig. 7 Comparison of the proposed R_D spectra with the previous R_μ & R_D spectra

6.2 Comparison with the existing strength reduction factor

Some scholars have indicated that the relational expression $R_\mu - \mu - T$ of the strength reduction factor are based on displacement ductility factor. In other words, the strength reduction factor is concerned with ductility factor and period (Newmark 1973, Nassar 1991, Miranda 1993). And the strength reduction factor considering cumulative damage is concerned with ductility factor, time period and damage index (Hou 2013, Lu 2008), expressed as $R_D - \mu - T - D$. In order to compare the difference between R_D and R_μ, the results of this paper and the existing strength reduction factors are listed in Fig.7. Under normal circumstances, R_μ and R_D changing within the structure period are the same. Under a single earthquake, the strength reduction factor R_D value based on the energy is less than the value of R_μ under the same conditions. Meanwhile, when the ductility factor is small, the difference between the two is not obvious, but with the increase of the ductility factor, the difference between the two becomes obvious. Comparing the strength reduction factor of sequence-type mainshocks and aftershocks, the difference between the R_D and R_μ is obvious, but the results of the (Hou 2013) model, which considers the cumulative damage, is closer to that of this paper.

7. CONCLUSIONS

In this paper, the actual mainshock-aftershock sequence-type ground motion records were divided according to the site classification of Chinese seismic design code, which were used in the elastic-plastic time history analysis of the SDOF systems. Using the Park-Ang model damage index, and considering the influence of the cumulative damage on the strength reduction factor, the basic characteristics of strength reduction factor were obtained. The authors offer the following conclusions:

(1) The R_D, which considers cumulative damage, is significantly different from the ductility-based R_μ, with a maximum difference of 40% under earthquake ground motion, that is, the structural strength demand considering cumulative damage is greater than that which only considers ductility;

(2) The R_D under the mainshock-aftershock sequence-type ground motions is always less than the R_D under single ground motions, with a difference of up to 22%. The structural design strength demand under the mainshock-aftershock sequence-type ground motion is greater than that under single ground motions.

(3) Based on the results of the regression analysis, a strength reduction factor spectrum empirical formula was established, which can be used to determine the structural strength demand in the structure seismic design and establish inelastic spectra for seismic design theory.

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